15. [Number Patterns]

Skill 15.1 Completing number patterns in table format by adding, subtracting or multiplying by the same number.

- Look at consecutive terms in the second row of the table.
- Find the number and operation used to get from one term to the next.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next term of the pattern.

Q. What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	−1	3	−27	81	−243	?

A. −1 , 3 , −27 , 81 , −243 , ?
× (−3)

Rule: Multiply each term by −3.
$−1 × (−3) = 3$
$3 × (−3) = −27$
$−27 × (−3) = 81$
$81 × (−3) = −243$
$−243 × (−3) = 729$

−1 , 3 , −27 , 81 , −243 , **729**

Note that the value of each term in the pattern is a multiple of 3 in increasing order. The signs are changing.

a) What is the value of the missing term in the pattern?

position	1	2	3	4	5
term	3	7	11	15	?

3 + 4 = 7, 7 + 4 = 11, 11 + 4 = 15, 15 + 4 = 19

b) What is the value of the missing term in the pattern?

position	1	2	3	4	5
term	1	6	11	16	?

c) What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	17	12	7	2	−3	?

d) What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	21	15	9	3	−3	?

e) What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	1	−5	25	−125	625	?

f) What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	2	−4	8	−16	32	?

g) What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	−10	20	−40	80	−160	?

h) What is the value of the missing term in the pattern?

position	1	2	3	4	5	6
term	2	−10	50	−250	1250	?

Skill 15.2   Completing number patterns by using changing values in the rule.

- Look at consecutive terms of the pattern.
- Find the operation used to get from one term to the next.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.
  Hints: Every number pattern is created by a rule involving numbers and operations. Counting numbers, even numbers and odd numbers have patterns themselves that can become part of the rule (see below).

Q. Complete the pattern:

$3 , 5 , 9 , 15 , 23 ,$ ____ , ____

A. $3 , 5 , 9 , 15 , 23 ,$ ____ , ____
$+2 \quad +4 \quad +6 \quad +8$

Rule: *Add 2, then 4, then 6, then 8, etc. The pattern is formed by adding consecutive even numbers.*
$23 + 10 = 33$
$33 + 12 = 45$
$3 , 5 , 9 , 15 , 23 , \underline{\textbf{33}} , \underline{\textbf{45}}$

a) Complete the pattern:

$11 , 13 , 16 , 20 ,$ ____ , ____
$+2 \quad +3 \quad +4 \quad +5 \quad +6$

$20 + 5 = 25 \qquad 25 + 6 = 31$

b) Complete the pattern:

$2 , 3 , 5 , 8 , 12 ,$ ____ , ____

c) Complete the pattern:

$3 , 5 , 9 , 15 ,$ ____ , ____

d) Complete the pattern:

$1 , 3 , 7 , 13 ,$ ____ , ____

e) Complete the pattern:

$0 , 3 , 9 , 18 , 30 ,$ ____ , ____

f) Complete the pattern:

$49 , 48 , 45 , 40 , 33 ,$ ____ , ____

g) Complete the pattern:

$1 , 4 , 8 , 13 , 19 ,$ ____ , ____

h) Complete the pattern:

$5 , 6 , 9 , 14 , 21 ,$ ____ , ____

i) Complete the pattern:

$1 , 4 , 10 , 19 ,$ ____ , ____

j) Complete the pattern:

$30 , 28 , 24 , 18 , 10 ,$ ____ , ____

Skill 15.3   Completing number patterns by adding or subtracting
the same positive number to integers.

MM5.2 11 **2** 2 33 44
MM6.1 1 **1** 22 33 44

- Look at consecutive terms of the pattern.
- Find the operation used to get from one term to the next.
  Hint: Every number pattern is created by adding or subtracting the same positive integer.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Q.   Complete the pattern:

$-25, -17, -9, -1, 7,$ ____ , ____

Note that the value of each term in the pattern is a increasing. Then find by how much.

A. $-25, -17, -9, -1, 7,$ ____ , ____
$+8 \quad +8 \quad +8 \quad +8$

Rule: *Add 8 to each term.*
$7 + 8 = 15$
$15 + 8 = 23$
$-25, -17, -9, -1, 7, \underline{\mathbf{15}}, \underline{\mathbf{23}}$

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a)   Complete the pattern:

$35, 20, 5, -10,$ ____ , ____
$-15 \quad -15 \quad -15 \quad -15 \quad -15$
$-10 - 15 = -25 \qquad -25 - 15 = -40$

b)   Complete the pattern:

$8, 5, 2, -1, -4,$ ____ , ____

c)   Complete the pattern:

$-20, -14, -8, -2, 4,$ ____ , ____

d)   Complete the pattern:

$-16, -11, -6, -1, 4,$ ____ , ____

e)   Complete the pattern:

$9, 5, 1, -3, -7,$ ____ , ____

f)   Complete the pattern:

$10, 7, 4, 1, -2,$ ____ , ____

g)   Complete the pattern:

$-35, -28, -21, -14, -7,$ ____ , ____

h)   Complete the pattern:

$-19, -15, -11, -7, -3,$ ____ , ____

i)   Complete the pattern:

$16, 10, 4, -2, -8,$ ____ , ____

j)   Complete the pattern:

$12, 7, 2, -3, -8,$ ____ , ____

k)   Complete the pattern:

$46, 34, 22, 10, -2,$ ____ , ____

l)   Complete the pattern:

$-20, -11, -2, 7, 16,$ ____ , ____

- Look at consecutive terms of the pattern.
- Find the number and operation (in this case multiplication) used to get from one term to the next.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Q. Complete the pattern:

$-5 , 15 , -45 , 135 ,$ [____ , ____]

Note that the value of each term in the pattern is increasing, but the signs are changing.

A. $-5 , \quad 15 , \quad -45 , \quad 135 , \quad$ ____ , ____

$\times (-3) \quad \times (-3) \quad \times (-3)$

Rule: *Multiply each term by* -3.

$135 \times (-3) = -405$

$-405 \times (-3) = 1215$

$-5 , \quad 15 , \quad -45 , \quad 135 , \quad \underline{\boldsymbol{-405}} , \quad \underline{\boldsymbol{1215}}$

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a) Complete the pattern:

$1 , 3 , 9 , 27 , 81 ,$ [____ , ____]

$\times 3 \quad \times 3 \quad \times 3 \qquad \times 3 \qquad \times 3$

81 × 3 = 243   *243 × 3 = 729*

b) Complete the pattern:

$3 , 6 , 12 , 24 , 48 ,$ [____ , ____]

48 × 2 =

c) Complete the pattern:

$2 , 6 , 18 , 54 , 162 ,$ [____ , ____]

d) Complete the pattern:

$1 , 5 , 25 , 125 ,$ [____ , ____]

e) Complete the pattern:

$\dfrac{1}{36} , \dfrac{1}{6} , 1 , 6 ,$ [____ , ____]

$\times 6 \quad \times 6 \quad \times 6$

f) Complete the pattern:

$\dfrac{1}{25} , \dfrac{1}{5} , 1 , 5 ,$ [____ , ____]

g) Complete the pattern:

$\dfrac{5}{2} , 5 , 10 , 20 , 40 ,$ [____ , ____]

h) Complete the pattern:

$\dfrac{1}{49} , \dfrac{1}{7} , 1 , 7 , 49 ,$ [____ , ____]

i) Complete the pattern:

$2 , -4 , 8 , -16 , 32 ,$ [____ , ____]

j) Complete the pattern:

$1 , -4 , 16 , -64 ,$ [____ , ____]

k) Complete the pattern:

$-3 , 9 , -27 , 81 ,$ [____ , ____]

l) Complete the pattern:

$4 , -20 , 100 , -500 ,$ [____ , ____]

- Look at consecutive terms of the pattern.
- Find the number and operation (in this case division) used to get from one term to the next.
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Q. Complete the pattern:

$-1215 , 405 , -135 , 45 ,$ ____ , ____

NB: The value of each term in the pattern is decreasing, but the signs are changing.

A. $-1215 , 405 , -135 , 45 ,$ ____ , ____

$\div (-3) \quad \div (-3) \quad \div (-3)$

Rule: *Divide each term by -3.*

$45 \div (-3) = -9$

$-9 \div (-3) = 3$

$-1215 , 405 , -135 , 45 , \underline{-9} , \underline{3}$

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a) Complete the pattern:

$288 , -144 , 72 , -36 , 18 ,$ ____ , ____

$\div(-2) \; \div(-2) \; \div(-2) \; \div(-2) \quad \div(-2) \quad \div(-2)$

$18 \div (-2) = -9 \qquad -9 \div (-2) = 4.5$

b) Complete the pattern:

$1458 , 486 , 162 , 54 , 18 ,$ ____ , ____

c) Complete the pattern:

$3125 , 625 , 125 , 25 ,$ ____ , ____

d) Complete the pattern:

$1600 , 800 , 400 , 200 , 100 ,$ ____ , ____

e) Complete the pattern:

$-200\,000 , 20\,000 , -2000 , 200 ,$ ____ , ____

f) Complete the pattern:

$310\,000 , -31\,000 , 3100 , -310 ,$ ____ , ____

g) Complete the pattern:

$-6250 , 1250 , -250 , 50 ,$ ____ , ____

h) Complete the pattern:

$-64 , 32 , -16 , 8 ,$ ____ , ____

i) Complete the pattern:

$112 , 56 , 28 , 14 , 7 ,$ ____ , ____

j) Complete the pattern:

$54 , 18 , 6 , 2 ,$ ____ , ____

k) Complete the pattern:

$375 , 75 , 15 , 3 ,$ ____ , ____

l) Complete the pattern:

$7203 , 1029 , 147 , 21 , 3 ,$ ____ , ____

Skill 15.6 Finding a random term in a number pattern.

- Draw a table and list the given terms and the position each term occupies in the pattern.
- Look for a relationship between consecutive terms and/or between the term and its position in the pattern.
- Based on this relationship, find the requested term in the pattern.

Q. Find the 15th term in the pattern
5 , 7 , 9 , 11 , 13 , ...

A.

position	1st	2nd	3rd	4th	5th	.....	15th
term	5	7	9	11	13		?
relationship	1×2+3	2×2+3	3×2+3	4×2+3	5×2+3		15×2+3

$\times 2 + 3$

Relationship:
each term = twice its position plus 3
The 15th term of the pattern is:
15 × 2 + 3 = **33**

a) Find the 30th term in the pattern
2 , 4 , 6 , 8 , 10 , ...

position	1st	2nd	3rd	4th	5th	.....	30th
term	2	4	6	8	10		?
relationship	1 × 2	2 × 2	3 × 2	4 × 2	5 × 2		30 × 2

$\times 2$

Relationship: term =

30th term =   =

b) Find the 18th term in the pattern
5 , 10 , 15 , 20 , 25 , ...

position	1st	2nd	3rd	4th	5th	.....	18th
term	5	10					?
relationship							

Relationship: term =

18th term =   =

c) Find the 20th term in the pattern
8 , 13 , 18 , 23 , 28 , ...

position	1st	2nd	3rd	4th	5th	.....	20th
term							?
relationship							

Relationship: term =

20th term =   =

d) Find the 25th term in the pattern
4 , 6 , 8 , 10 , 12 , ...

position	1st	2nd	3rd	4th	5th	.....	25th
term							?
relationship							

Relationship: term =

25th term =   =

e) Find the 20th term in the pattern
1 , 4 , 7 , 10 , 13 , ...

position	1st	2nd	3rd	4th	5th	.....	20th
term							?
relationship							

Relationship: term =

20th term =   =

f) Find the 8th term in the pattern
1 , 2 , 4 , 8 , 16 , ...

position	1st	2nd	3rd	4th	5th	.....	8th
term							?
relationship							

Relationship: term =

8th term =   =

MM5.2 1 1 2 2 3 3 **4** 4
MM6.1 1 1 2 2 3 **3** 4 4

- Substitute the value of n in the formula for the general rule of the sequence.
- Calculate the value of the required term.

Q. If the general rule of a sequence is $\frac{n}{4} - 9$ find the 60th term ($n = 60$).

A. *the nth term of the sequence =*

$$= \frac{n}{4} - 9 \quad \text{⟵ substitute } n = 60$$

the 60th term of the sequence =

$$= \frac{60}{4} - 9$$

$$= 15 - 9$$

$$= \boldsymbol{6}$$

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a) If the general rule of a sequence is $5n - 4$ find the 30th term ($n = 30$).

30th term = 5 × 30 − 4

= 150 − 4   = ☐

b) If the general rule of a sequence is $4n - 7$ find the 15th term ($n = 15$).

15th term =

=   = ☐

c) If the general rule of a sequence is $8 - 5n$ find the 10th term ($n = 10$).

10th term =

=   = ☐

d) If the general rule of a sequence is $25 - n$ find the 40th term ($n = 40$).

40th term =

=   = ☐

e) If the general rule of a sequence is $15n$ find the 30th term ($n = 30$).

30th term =

=   = ☐

f) If the general rule of a sequence is $-40n$ find the 25th term ($n = 25$).

25th term =

=   = ☐

g) If the general rule of a sequence is $-\frac{2n}{7}$ find the 35th term.

35th term =

=   = ☐

h) If the general rule of a sequence is $\frac{n}{3} + 1$ find the 21st term.

21st term =

=   = ☐

i) If the general rule of a sequence is $-6(n - 3)$ find the 23rd term.

23rd term =

=   = ☐

j) If the general rule of a sequence is $3(n - 6)$ find the 24th term.

24th term =

=   = ☐

Skill 15.8   Finding the general rule of a pattern given a table of values for the pattern (1).

MM5.2 1 1 2 2 3 3 4 **4**
MM6.1 1 1 2 2 3 3 **4** 4

To decide which general formula is true for the values shown in the table
- Substitute the values for *n* (first row in the table) in the general formula.
- Check if the results match the term values given in the second row of the table.

To express a general term of a sequence

EITHER
- Look for a relationship between consecutive terms and/or between the term and its position in the sequence.

  Hints: This relationship is an expression of n involving one or more operations, i.e. n – 6, 2n, 4n + 2

OR
- Find the difference between consecutive terms of the sequence (common difference).
- Write the term "common difference × *n*" in the expression.
- Check the result by substituting a random value for *n* into the formula.
- Adjust the expression by adding, subtracting, multiplying or dividing by a constant.
- Check the result by substituting all values for *n* = 1, 2, 3, 4, 5 into the final formula.

Q. Write an expression for the term in position *n* given the table of values for the sequence.

position	1	2	3	4	5	....	*n*
term	1	5	9	13	17	....	

+4 +4 +4 +4

A. *common difference = 4*

$\Rightarrow$ *term in position n contains 4n*

If term in position n = 4n and n = 1
$\Rightarrow$ *term 1 = 4 × 1 = 4 ≠ 1* — Adjust by subtracting 3

If term in position n = 4n – 3
n = 1 ⇒ term 1 = 4 × 1 – 3 = 1 (true)
n = 2 ⇒ term 2 = 4 × 2 – 3 = 5 (true)
n = 3 ⇒ term 3 = 4 × 3 – 3 = 9 (true)
n = 4 ⇒ term 4 = 4 × 4 – 3 = 13 (true)
n = 5 ⇒ term 5 = 4 × 5 – 3 = 17 (true)

The term in position n is **4n – 3**

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a)

position	1	2	3	4	5	....	*n*
term	7	9	11	13	15	....	?

The rule for the term in position *n* is:
A) 5*n* + 2   B) 2*n* – 5
C) 5*n* + 5   D) 2*n* + 5

A n = 1 ⇒ term 1 = 5 × 1 + 2 = 7 ⇒ true

n = 2 ⇒ term 2 = 5 × 2 + 2 = 12 ⇒ false

B n = 1 ⇒ term 1 = 2 × 1 – 5 = –3 ⇒ false

C n = 1 ⇒ term 1 = 5 × 1 + 5 = 10 ⇒ false

D n = 1 ⇒ term 1 = 2 × 1 + 5 = 7 ⇒ true

n = 2 ⇒ term 2 = 2 × 2 + 5 = 9 ⇒ true

n = 3 ⇒ term 3 = 2 × 3 + 5 = 11 ⇒ true

n = 4 ⇒ term 4 = 2 × 4 + 5 = 13 ⇒ true

n = 5 ⇒ term 5 = 2 × 5 + 5 = 15   **D**

b)

position	1	2	3	4	5	....	*n*
term	7	10	13	16	19	....	?

The rule for the term in position *n* is:
A) 3*n* – 4   B) 3*n* + 4
C) 4*n* + 3   D) 4*n* – 3

A

Skill 15.8 Finding the general rule of a pattern given a table of values for the pattern (2).

MM5.2 11 22 33 4**4**
MM6.1 11 22 33 4**4**

c) Write an expression for the term in position n given the table of values for the sequence.

position	1	2	3	4	5	….	n
term	8	10	12	14	16	….	$2n + 6$

$+2 \quad +2 \quad +2 \quad +2$

common difference = 2

term n = 2n ⇒ term 1 = 2 × 1 = 2 (false)

adjust *term n = 2n + 6 ⇒ term 1 = 8 (true)*

d) Write an expression for the term in position n given the table of values for the sequence.

position	1	2	3	4	5	….	n
term	5	4	3	2	1	….	

$-1 \quad -1 \quad -1 \quad -1$

common difference = −1

term n = −n ⇒ term 1 = −1 (false)

adjust *term n = 6 − n ⇒ term 1 = 5 (true)*

e) Write an expression for the term in position n given the table of values for the sequence.

position	1	2	3	4	5	….	n
term	2	5	8	11	14	….	

common difference =

term n

adjust *term n*

f) Write an expression for the term in position n given the table of values for the sequence.

position	1	2	3	4	5	….	n
term	5	9	13	17	21	….	

common difference =

term n

adjust *term n*

g) Write an expression for the term in position n given the table of values for the sequence.

position	1	2	3	4	5	….	n
term	5	10	15	20	25	….	

common difference =

term n

adjust *term n*

h) Write an expression for the term in position n given the table of values for the sequence.

position	1	2	3	4	5	….	n
term	7	10	13	16	19	….	

common difference =

term n

adjust *term n*

i) Write an expression for the term in position n given the table of values for the sequence.

position	1	2	3	4	5	….	n
term	2	1	0	−1	−2	….	

common difference =

term n

adjust *term n*

j) Write an expression for the term in position n given the table of values for the sequence.

position	1	2	3	4	5	….	n
term	−3	−6	−9	−12	−15	….	

common difference =

term n

adjust *term n*

- Look at consecutive terms of the pattern.
- Find the operation used to get from one term to the next.
  Hint: Every number pattern is created by adding, subtracting, multiplying or dividing by rational numbers (whole numbers, fractions or decimals).
- Define the rule (operation) of the pattern.
- Apply this rule to the last given term and find the next two terms of the pattern.

Q. Complete the pattern:

$$\frac{5}{18}, \frac{1}{2}, \frac{13}{18}, \frac{17}{18}, 1\frac{1}{6}, \boxed{\quad,\quad}$$

A. $\frac{5}{18}, \frac{1}{2}, \frac{13}{18}, \frac{17}{18}, 1\frac{1}{6},$

Look at the 3rd and 4th terms: their difference is $\frac{4}{18}$

Rule: *Add* $\frac{4}{18}$ *to each term.*

$$1\frac{1}{6} + \boxed{\frac{4}{18}} = \frac{7}{6} + \frac{4}{18} = \frac{21}{18} + \frac{4}{18} = \frac{25}{18} = 1\frac{7}{18}$$

$$\frac{25}{18} + \boxed{\frac{4}{18}} = \frac{29}{18} = 1\frac{11}{18}$$

$$\frac{5}{18}, \frac{1}{2}, \frac{13}{18}, \frac{17}{18}, 1\frac{1}{6}, \mathbf{1\frac{7}{18}}, \mathbf{1\frac{11}{18}}$$

a) Complete the pattern:

$$1, 2, 3.5, 5.5, 8, \boxed{\quad,\quad}$$
$$+1 \ +1.5 \ +2 \ +2.5 \ \ +3 \ \ +3.5$$

$$8 + 3 = 11 \qquad 11 + 3.5 = 14.5$$

b) Complete the pattern:

$$0.8, 2, 3.4, 5, 6.8, \boxed{\quad,\quad}$$
$$+1.2$$

c) Complete the pattern:

$$1.5, 3.5, 6, 9, 12.5, \boxed{\quad,\quad}$$

d) Complete the pattern:

$$4, 5.5, 7.5, 10, 13, \boxed{\quad,\quad}$$

e) Complete the pattern:

$$1.75, 3.5, 7, 14, \boxed{\quad,\quad}$$

f) Complete the pattern:

$$1\frac{1}{4}, 2\frac{1}{2}, 5, 10, \boxed{\quad,\quad}$$

g) Complete the pattern:

$$36, 18, 9, \frac{9}{2}, \frac{9}{4}, \boxed{\quad,\quad}$$

h) Complete the pattern:

$$32, 8, 2, \frac{1}{2}, \frac{1}{8}, \boxed{\quad,\quad}$$

i) Complete the pattern:

$$3\frac{1}{4}, 4, 4\frac{3}{4}, 5\frac{1}{2}, 6\frac{1}{4}, \boxed{\quad,\quad}$$

j) Complete the pattern:

$$\frac{8}{15}, \frac{4}{5}, 1\frac{1}{15}, 1\frac{1}{3}, 1\frac{3}{5}, \boxed{\quad,\quad}$$