19. [Factorisation]

Skill 19.1 Factorising by finding the HCF of the coefficients.

MM5.2 **1 1** 2 2 3 3 4 4
MM6.1 **1** 1 2 2 3 3 4 4

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Write the HCF in front of the brackets.
- Divide each term by the HCF to find the remaining factors.
- Write the remaining factors inside the brackets.
- Keep the signs.

Q. Factorise $15a - 24$

A. *15a and 24*
3 × 5 = 15 and 3 × 8 = 24
HCF of 15 and 24 is 3
15a ÷ 3 = 5a and 24 ÷ 3 = 8
Remaining factors are 5a and 8
15a – 24 = ***3(5a – 8)***

Write the HCF before the ( ) — *Keep the sign*

a) Factorise $4k - 16$ = $4(k - 4)$

4 × k = 4k and 4 × 4 = 16 ⇒ HCF is 4

b) Factorise $4x + 8$ = ...

c) Factorise $6s + 18$ = ...

d) Factorise $3u - 15$ = ...

e) Factorise $9m - 2$ = ...

f) Factorise $14n + 21$ = ...

g) Factorise $2y + 10z$ = ...

h) Factorise $4a - 12b$ = ...

i) Factorise $6d + 14e$ = ...

j) Factorise $16uv - 4$ = ...

k) Factorise $12k - 8l$ = ...

l) Factorise $4g + 4h - 6$ = ...

m) Factorise $3m - 6n + 9$ = ...

n) Factorise $10v - 5w + 15$ = ...

o) Factorise $5h^2 - 10i + 25j$ = ...

p) Factorise $6r^2 - 27s + 9t$ = ...

Skill 19.2 Factorising by finding the HCF of coefficients and variables.

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the variables in each term.
- Write the HCF and any other CF in front of the brackets.
- Divide each term by any common factors to find the remaining factors.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

Q. Factorise $18kl - 24k$

A. *18kl and 24k*
6 × 3 = 18 and 6 × 4 = 24
HCF of 18 and 24 is 6
k is common to both terms CF is k
18kl ÷ 6k = 3l and 24k ÷ 6k = 4
Remaining factors are 3l and 4
Write all CF's before the ( )
$18kl - 24k = \mathbf{6k(3l - 4)}$ *Keep the sign*

a) Factorise $ab + 5b$ *ab = ba*

b is common to both terms = $b(a + 5)$

b) Factorise $de + d$

..... = ...

c) Factorise $7e + ef$

..... = ...

d) Factorise $3st + 4s$

..... = ...

e) Factorise $8ab - 4b$

..... = ...

f) Factorise $15g + 20gh$

..... = ...

g) Factorise $wx - xy$

..... = ...

h) Factorise $2jk + 2kl$

..... = ...

i) Factorise $uv - 3vw$

..... = ...

j) Factorise $8ab + 4bc$

..... = ...

k) Factorise $12qr + 8rs$

..... = ...

l) Factorise $15de - 6ef$

..... = ...

m) Factorise $10cd - 8d$

..... = ...

n) Factorise $15m - 10mn$

..... = ...

o) Factorise $21qr + 14pq$

..... = ...

p) Factorise $6tu + 18uv$

..... = ...

q) Factorise $6xy + 9yz$

..... = ...

r) Factorise $10gh - 25gi$

..... = ...

Skill 19.3 Factorising to simplify expressions involving large numbers.

- Find the number repeating in both products.
- Write this number in front of the brackets.
  Hint: When both terms are negative the negative sign is taken out as a common factor.
- Write the remaining factors inside the brackets.
- Keep the signs.

Q. Factorise and evaluate
$45 \times 7 + 45 \times 3$

A. $45 \times 7 + 45 \times 3$ *45 is repeating*
$= 45 \times (7 + 3)$
$= 45 \times 10$
$= \mathbf{450}$

a) Factorise and evaluate
$99 \times 99 - 98 \times 99$

$= 99 \times (99 - 98)$
$= 99 \times 1$ $=$ 99

b) Factorise and evaluate
$15 \times 14 + 15 \times 6$

$= 15 \times (14 + 6)$
$=$ ..... $=$

c) Factorise and evaluate
$987 \times 2 + 987 \times 8$

$=$ .....
$=$ ..... $=$

d) Factorise and evaluate
$40 \times 8 + 40 \times 12$

$=$ .....
$=$ ..... $=$

e) Factorise and evaluate
$23 \times 37 + 23 \times 63$

$=$ .....
$=$ ..... $=$

f) Factorise and evaluate
$25 \times 26 + 25 \times 24$

$=$ .....
$=$ ..... $=$

g) Factorise and evaluate
$999 \times 9 - 999 \times 8$

$=$ .....
$=$ ..... $=$

h) Factorise and evaluate
$87 \times 19 - 87 \times 9$

$=$ .....
$=$ ..... $=$

i) Factorise and evaluate
$-4 \times 14 - 4 \times 6$

Both terms are negative so CF is negative

− + = −

$= -4 \times (14 + 6)$
$=$ ..... $=$

j) Factorise and evaluate
$-9 \times 33 - 9 \times 67$

$=$ .....
$=$ ..... $=$

Skill 19.4 Factorising involving squared terms.

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the variables in each term.
- Write the HCF and any other CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Check the signs.

Q. Factorise $2wx - 12w^2x$

A. *2wx and 12w²x*
2 × 1 = 2 and 2 × 6 = 12
HCF of 2 and 12 is 2
wx is common to both terms CF is wx
Remaining factors are 1 and 6w
Write all CF's before the ( )
$2wx - 12w^2x = \mathbf{2wx(1 - 6w)}$
Keep the sign

a) Factorise $2j^2k + 5j$

$CF = j$ = $j(2jk + 5)$

Write all CF's outside the ( )

b) Factorise $e^2 + 7e$

..... =

c) Factorise $h + 4h^2$

..... =

d) Factorise $m^2 - 9m$

..... =

e) Factorise $3c - 12c^2$

..... =

f) Factorise $4f^2 + 6f$

..... =

g) Factorise $fg^2 + f$

..... =

h) Factorise $10b - 16ab^2$

..... =

i) Factorise $p^2q - 3p$

..... =

j) Factorise $12i - 18hi^2$

..... =

k) Factorise $14bc + 2b^2c$

..... =

l) Factorise $5r^2s - r^2t$

..... =

m) Factorise $vw + 7v^2 - 3vwx$

..... =

n) Factorise $8j^2 - 24jk + 12jl$

..... =

o) Factorise $f^3g^2 + fg^2$

..... =

p) Factorise $p^3q^2 + p^2q + pq$

..... =

Skill 19.5 Factorising negative terms.

- Find the highest common factor (HCF) of the coefficients in each term. (see skill 5.1, page 49)
- Find any common factors (CF) from the variables in each term.
- Write the HCF and any other CF in front of the brackets.
  Hint: When both terms are negative the negative sign is taken out as a common factor.
- Write the remaining factors inside the brackets.
- Check the signs.

Q. Factorise $-10r^2 - 5r$

A. *$-10r^2$ and $5r$*
$5 \times 2 = 10$ and $5 \times 1 = 5$
HCF of 10 and 5 is 5
r is common to both terms CF is r
$-$ is common to both terms CF is –
Remaining factors are r and 1
Write all CF before the ( )
$-10r^2 - 5r = \mathbf{-5r(r + 1)}$

"–" is common to both terms

a) Factorise $-7a - 21$ = $-7(a + 3)$

$7 \times 1 = 7$ and $7 \times 3 = 21$ $\Rightarrow$ HCF is 7

b) Factorise $-4k - 12$ =

$4 \times 1 = 4$ and $4 \times 3 = 12$ $\Rightarrow$ HCF is

c) Factorise $-6g - 15$ =

d) Factorise $-6e - 14$ =

e) Factorise $-2h^2 - 6h$ =

f) Factorise $-8z^2 - 28z$ =

g) Factorise $-12i^3 - 9ij$ =

h) Factorise $-t^3 - 5t^2u$ =

i) Factorise $-6bc^2 + 3c^2$ =

j) Factorise $-5x^2y^2 - xy^3$ =

k) Factorise $-2x^3 - 4xy$ =

l) Factorise $-4m^3 - 12mn^2 + 18m$ =

m) Factorise $-2k^3 + 6k^3l + 8k$ =

n) Factorise $-2hi^3 + 3h^2i - 5h^2$ =

Skill 19.6 Factorising by finding binomial factors.

- Find any common factors (CF).
  Hint: It might help to think of common factors that are expressions like (d + 2) as a blob ■.
- Write the CF in front of the brackets.
- Write the remaining factors inside the brackets.
- Keep the signs.
- Check the result by expanding the brackets.

Q. Factorise $2(r - 1) - r(r - 1)$

A. $2(r - 1) - r(r - 1)$ *Consider (r – 1) = ■*
$= 2\blacksquare - r\blacksquare$ *Keep the sign*
$= \blacksquare(2 - r)$
$= \boldsymbol{(r - 1)(2 - r)}$

a) Factorise
$d(d + 2) + 8(d + 2)$ *Consider (d + 2) = ■*
$= d\blacksquare + 8\blacksquare$ *Keep the sign*
$= \blacksquare(d + 8)$ $= (d + 2)(d + 8)$

b) Factorise
$2(h - 3) + h(h - 3)$
$= 2\blacksquare + h\blacksquare$
$=$ $=$

c) Factorise
$5(x + 4) + x(x + 4)$
$=$
$=$ $=$

d) Factorise
$b(b - 7) + 6(b - 7)$
$=$
$=$ $=$

e) Factorise
$a(a + 2) - 9(a + 2)$
$=$
$=$ $=$

f) Factorise
$z(z - 5) - (z - 5)$
$=$
$=$ $=$

g) Factorise
$j(j + 4) + j + 4$
$=$
$=$ $=$

h) Factorise
$m(n - 2) + 4(n - 2)$
$=$
$=$ $=$

i) Factorise
$3x(2x - 5) - 4(2x - 5)$
$=$
$=$ $=$

j) Factorise
$d(c + 5) - (c + 5)$
$=$
$=$ $=$

k) Factorise
$q(s - 3) + t(s - 3)$
$=$
$=$ $=$

l) Factorise
$6v(2w - 1) + 4(2w - 1)$
$=$
$=$ $=$

Skill 19.7 Factorising four terms by grouping 2 and 2.

- Begin factorising by grouping the 4 terms in 2 groups of 2.
- Take out the CF from the first group of 2 and write it in front of the brackets.
- Take out the CF from the second group of 2 and write it in front of the brackets.
- Keep the signs.
- Factorise again by finding the common binomial factor. (see skill 19.6, page 198)
  Hint: It might help to think of binomial factors that are expressions like (d + 2) as a blob ■.
- Take out the binomial factor or blob and write it in front of the brackets.
- Write the remaining factors inside the brackets.
- Check the result by expanding the brackets.

Q. Factorise $m^2 + 3m + 5m + 15$

A. $m^2 + 3m + 5m + 15$ — *Keep the sign*

$= m(m + 3) + 5(m + 3)$

$= m(\blacksquare) + 5(\blacksquare)$ — *Consider (m + 3) = ■*

$= \blacksquare(m + 5)$

$= \boldsymbol{(m + 3)(m + 5)}$

a) Factorise $c^2 + 8c + 3c + 24$ — *Group 2 and 2, Factorise each group*

$= c(c + 8) + 3(c + 8)$ — *Factorise again, Consider (c + 8) = ■*

$= \blacksquare(c + 3)$ = $(c + 8)(c + 3)$

b) Factorise $a^2 + 3a + 2a + 6$

$= a(a + 3) + 2(a + 3)$

$=$ ..... $=$ .....

c) Factorise $s^2 + 6s + 5s + 30$

$=$ .....

$=$ ..... $=$ .....

d) Factorise $h^2 + 5h + 4h + 20$

$=$ .....

$=$ ..... $=$ .....

e) Factorise $v^2 + 7v + 3v + 21$

$=$ .....

$=$ ..... $=$ .....

f) Factorise $4n + n^2 + 16 + 4n$

$=$ .....

$=$ ..... $=$ .....

g) Factorise $6t + t^2 - 42 - 7t$

$=$ .....

$=$ ..... $=$ .....

h) Factorise $4b + 4 - b^2 - b$

$=$ .....

$=$ ..... $=$ .....

i) Factorise $5p - 10 + p^2 - 2p$

$=$ .....

$=$ ..... $=$ .....

j) Factorise $q^2 + 5q - 4q - 20$

$=$ .....

$=$ ..... $=$ .....

Skill 19.8 Factorising using the difference of perfect squares.

- Find any common factors (CF) of the terms.
- Write any CF in front of the brackets.
- Use the difference of perfect squares formula.
- Check the result by expanding the brackets.

Expand the brackets

$(a + b)(a - b) = a \times a + a \times -b + b \times a + b \times -b$

$= a^2 - ab + ba - b^2$ (① ② ③ ④)

$= a^2 - b^2$ *Group like terms*

Q. Factorise $5w^2 - 20$

A. $5w^2 - 20$ *Take out the CF of 5*

$= 5(w^2 - 4)$ *$4 = 2^2$*

$= 5(w^2 - 2^2)$ *Use $a^2 - b^2 = (a + b)(a - b)$ where $a = w$ and $b = 2$*

$= \mathbf{5(w + 2)(w - 2)}$

a) Factorise $c^2 - 81$ *$b^2 = 81$, $b = 9$*

Use $a^2 - b^2 = (a + b)(a - b)$

$= c^2 - 9^2$ $=$ $(c + 9)(c - 9)$

b) Factorise $y^2 - 4$

$=$ ..... $=$ .....

c) Factorise $d^2 - e^2$

$=$ ..... $=$ .....

d) Factorise $36 - h^2$

$=$ ..... $=$ .....

e) Factorise $4j^2 - 9$

$=$ ..... $=$ .....

f) Factorise $2c^2 - 50$

$=$ ..... $=$ .....

g) Factorise $p^2 - 81q^2$

$=$ ..... $=$ .....

h) Factorise $80 - 5y^2$

$=$ ..... $=$ .....

i) Factorise $9a^2 - 36b^2$

$=$ ..... $=$ .....

j) Factorise $75 - 3z^2$

$=$ ..... $=$ .....

k) Factorise $3d^2 - 27$

$=$ ..... $=$ .....

l) Factorise $100 - 4k^2$

$=$ ..... $=$ .....

Skill 19.9 Factorising quadratic trinomials.

- Write two sets of brackets. Because x^2 can only be produced from $x \times x$, write the factors of the squared pronumeral in the brackets $(x \quad)(x \quad)$.
- Make a list of all pairs of factors, positive and negative, that produce the whole number.
- From this list determine which pair can be added to get the correct number of x terms.
- Write the result in the brackets with their signs.
- Check the result by expanding the brackets.

Q. Factorise $x^2 - 9x + 8$

A. $x^2 - 9x + 8$

$= (x \quad)(x \quad)$ — *Write x in the brackets*

$8 = 1 \times 8 = -1 \times -8 = 2 \times 4 = -2 \times -4$ — *List the factors of +8*

Only $-1x$ and $-8x$ can make $-9x$ — *Determine the x terms*

$= x^2 - 1x - 8x + 8$

$= \mathbf{(x - 1)(x - 8)}$

CHECK — *Expand the brackets*

$= (x - 1)(x - 8)$

$= x^2 - 1x - 8x + 8$

$= x^2 - 9x + 8 = \mathbf{(x - 1)(x - 8)}$ ✓

AND/OR consider

$= x^2 - 1x - 8x + 8$ — *Group 2 and 2*

$= x(x - 1) - 8(x - 1)$ — *Factorise each group*

$= x(\blacksquare) - 8(\blacksquare)$

$= \blacksquare(x - 8)$ — *Factorise again. Consider $(x - 1) = \blacksquare$*

$= \mathbf{(x - 1)(x - 8)}$

a) Find the missing factor
$x^2 + 7x + 10$

$10 = 2 \times 5 = -2 \times -5$

$5x + 2x = 7x$ $= (x + 5)(x + 2)$

b) Find the missing factor
$d^2 - 4d + 4$

$4 = 2 \times 2 = -2 \times -2$

$= (d - 2)(\quad)$

c) Find the missing factor
$s^2 + 4s + 3$

$= (s + 3)(\quad)$

d) Find the missing factor
$g^2 + 8g + 15$

$= (g + 5)(\quad)$

e) Factorise
$m^2 + 2m - 24$

Which pair can be added to get +2m?

$-24 = -4 \times 6 = 4 \times -6$

$6m - 4m = 2m$ $=$

f) Factorise
$j^2 + 11j + 24$

$=$

g) Factorise
$y^2 + 5y + 4$

$=$

h) Factorise
$z^2 - 6z + 8$

$=$

i) Factorise
$c^2 - 6c + 5$

$=$

j) Factorise
$p^2 - 6p - 16$

$=$