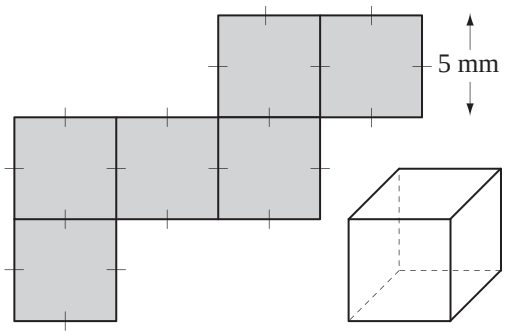
24. [Surface Area]

Skill 24.1 Calculating the total surface area (*TSA*) of rectangular prisms and cubes using nets (1).

MM5.2 **1** 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Find any unknown side lengths.
- Calculate the area of each face as shown on the net.
  Hint: Rectangular prisms have 6 faces of 3 different sizes: base and top (2)
  front and back (2)
  other faces (2)
- Add together the area of all faces.
  Hints: Sides marked with a dash (|) are of equal length.
  Sides marked with two dashes (||) are of equal length etc.

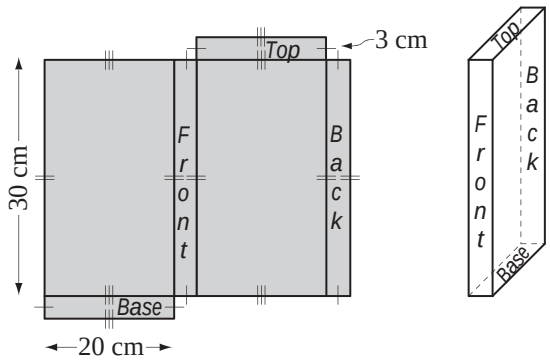
Q. Find the total surface area of the cube by finding the area of its net.



A. *Area of square face* $= 5\text{ mm} \times 5\text{ mm}$
$= 25\text{ mm}^2$

$TSA = 25\text{ mm}^2 \times 6$ (*A cube has 6 identical faces*)
$= \mathbf{150\ mm^2}$

a) Find the total surface area of the rectangular prism by finding the area of its net.



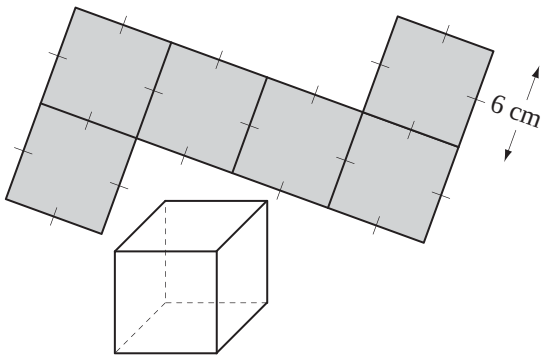
Area: base & top = 2 × 20 × 3 = 120

Area: front & back = 2 × 30 × 3 = 180

Area: 2 other faces = 2 × 30 × 20 = 1200

TSA = 120 + 180 + 1200 = ____ cm^2

b) Find the total surface area of the cube by finding the area of its net.

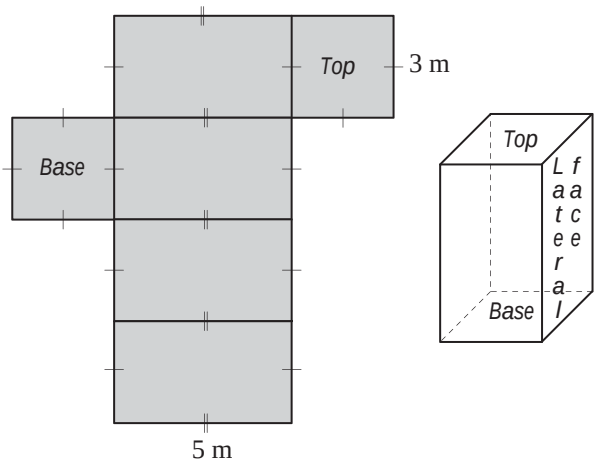


Area of 1 face = ____

TSA = ____ = ____ cm^2

Skill 24.1 Calculating the total surface area (*TSA*) of rectangular prisms and cubes using nets (2).

c) Find the total surface area of the square prism by finding the area of its net.

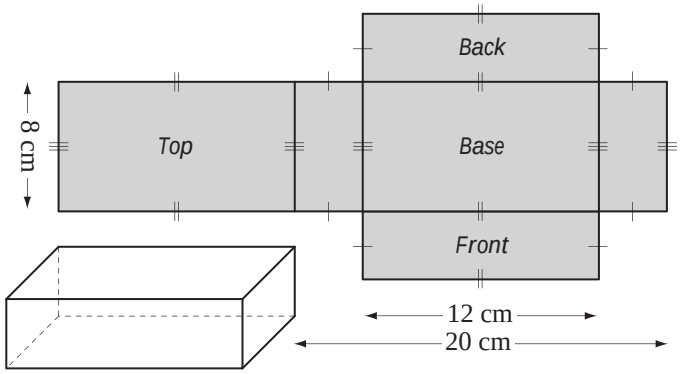


Area: base & top = ...

Area: 4 lateral faces = ...

TSA = ... = ... m^2

d) Find the total surface area of the rectangular prism by finding the area of its net.



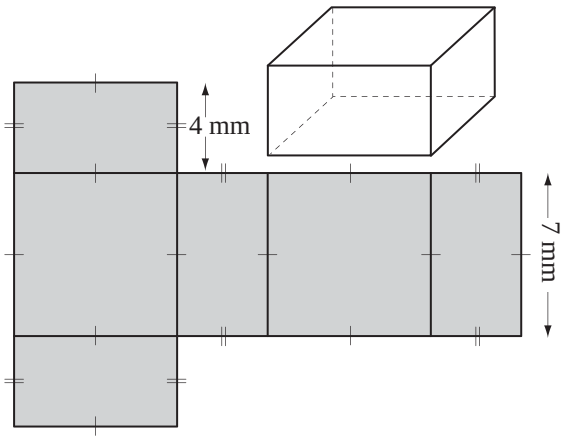
Area: base & top = ...

Area: front & back = ...

Area: 2 other faces = ...

TSA = ... = ... cm^2

e) Find the total surface area of the square prism by finding the area of its net.

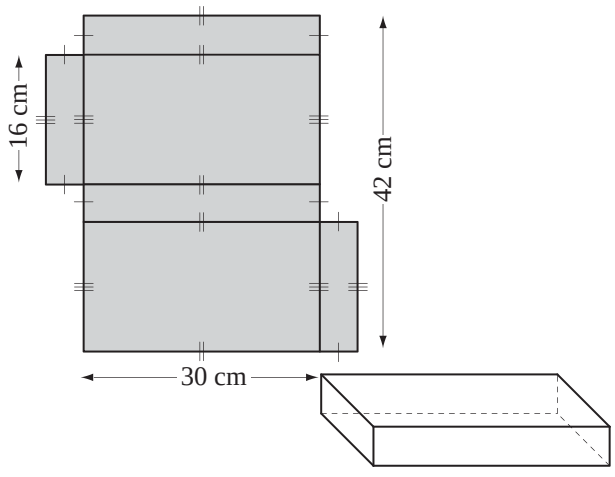


.....

.....

TSA = ... = ... mm^2

f) Find the total surface area of the rectangular prism by finding the area of its net.



.....

.....

.....

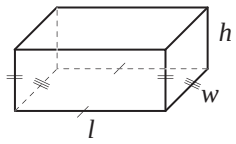
TSA = ... = ... cm^2

Skill 24.2 Calculating the total surface area (*TSA*) of rectangular prisms.

rectangular prism

TSA = 2(length × width) + 2(length × height) + 2(width × height)

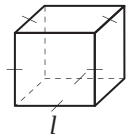
TSA = 2*lw* + 2*lh* + 2*wh* = 2(*lw* + *lh* + *wh*)



cube

TSA = 6(length × length)

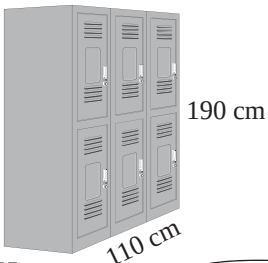
TSA = 6l^2



Q. Lewis wants to make a box, with a lid, for his card collection. The box needs a base of 11 cm by 20 cm and must be 12 cm high. How much wood does Lewis need?

A. *TSA* = 2 × (11 × 20 + 11 × 12 + 20 × 12)
= 2 × (220 + 132 + 240)
= 2 × 592
= ***1184 cm²***

a) The locker block needs to be resurfaced. What is the surface area of this rectangular prism disregarding its base?



Subtract 1 base area

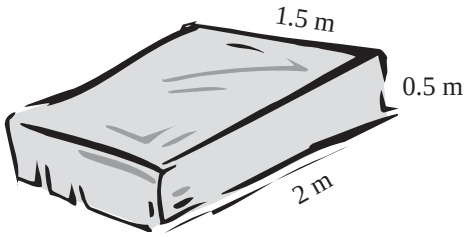
TSA = *lw* + 2*lh* + 2*wh*

= 110 × 55 + 2 × 110 × 190 + 2 × 55 × 190

= 6050 + 2 × 20 900 + 2 × 10 450

= 6050 + 41 800 + 20 900 = ____ cm²

b) Zoe's mattress was torn in removal. What is the minimum amount of mattress ticking needed to re-cover the mattress?



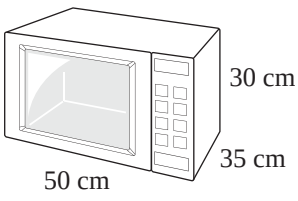
TSA = 2(*lw* + *lh* + *wh*)

= ____

= ____

= ____ = ____ m²

c) Find the total surface area of the microwave.



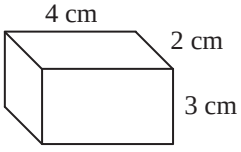
TSA = ____

= ____

= ____

= ____ = ____ cm²

d) The total surface area of the rectangular prism is 52 m². What is the *TSA* if all the dimensions are doubled?



TSA = ____

= ____

= ____

= ____ = ____ cm²

Skill 24.3 Calculating the total surface area (*TSA*) of rectangular composite solids (1).

- Find any unknown side lengths.
- Calculate the area of each face.
- Add together the area of all faces.

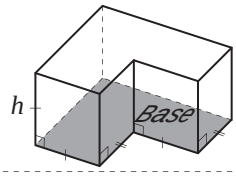
OR

- Identify the base by finding the two, identical parallel faces.
  Hint: A prism does not necessarily sit on its base.
- Substitute values into the formula:

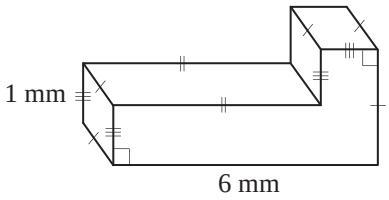
rectangular composite solid

TSA = Perimeter of base × height + 2 × Area of base

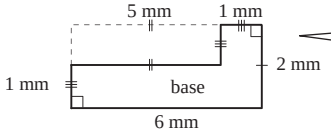
$TSA = P_b h + 2A_b$



Q. Find the total surface area of the prism.



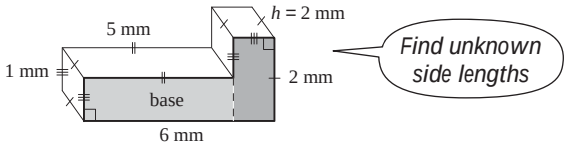
A.



$P_b = 6 + 1 + 5 + 1 + 1 + 2 = 16$

OR

$P_b = 6 + 6 + 2 + 2 = 16$



$A_b = 5 \times 1 + 2 \times 1$

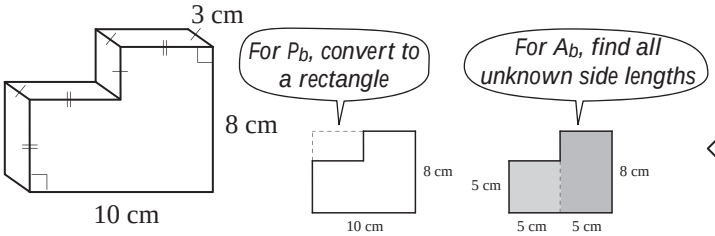
$= 5 + 2 = 7$

$TSA = P_b h + 2A_b$

$= 16 \times 2 + 2 \times 7$

$= 32 + 14 =$ **$46\ mm^2$**

a) Find the total surface area of the prism.



$P_b = 10 + 10 + 8 + 8 = 36$

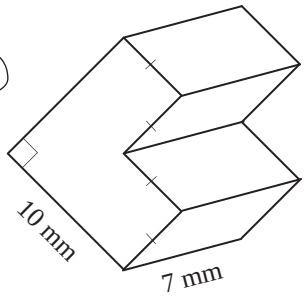
$A_b = 5 \times 5 + 5 \times 8 = 25 + 40 = 65$

$TSA = P_b h + 2A_b$ *Use TSA formula for a prism*

$= 36 \times 3 + 2 \times 65$

$= 108 + 130$ = ____ cm^2

b) Find the total surface area of the prism.



$P_b =$

$A_b =$

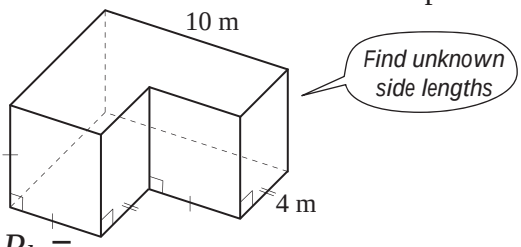
$TSA = P_b h + 2A_b$

=

= ____ mm^2

Skill 24.3 Calculating the total surface area (*TSA*) of rectangular composite solids (2).

c) Find the total surface area of the prism.



P_b =

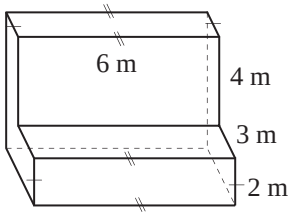
A_b =

$TSA = P_bh + 2A_b$

=

= = m^2

d) Find the total surface area of the prism.



P_b =

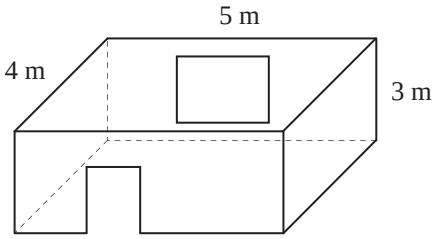
A_b =

$TSA = P_bh + 2A_b$

=

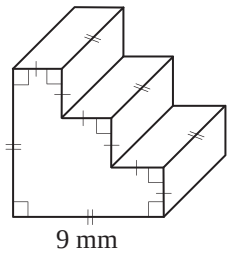
= = m^2

e) A window 2 m by 1.5 m and a doorway 2m by 0.8 m are in the plan for this room. Find the total area of the inside walls to be painted.



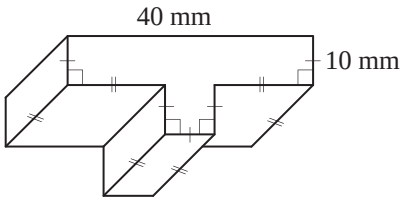
TSA = = m^2

f) Find the total surface area of the prism.



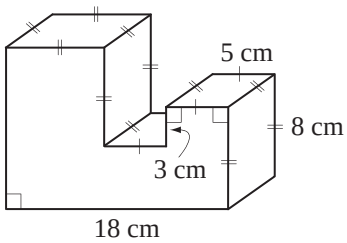
TSA = = mm^2

g) Find the total surface area of the prism.



TSA = = mm^2

h) Find the total surface area of the prism.



TSA = = cm^2

Skill 24.4 Calculating the total surface area (*TSA*) of triangular prisms (1).

- Find any unknown side lengths.
- Calculate the area of each face.
- Add together the area of all faces.

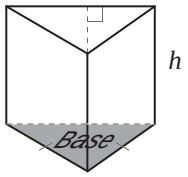
OR

- Substitute values into the formula:

triangular prism

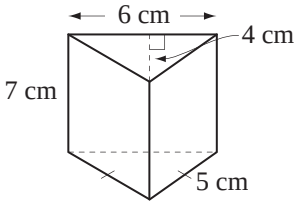
TSA = Perimeter of base × height + 2 × Area of base

$TSA = P_b h + 2A_b$

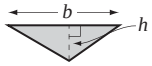


Hint: Do not confuse the height needed to calculate the area of the triangular base, with the height (h) of the prism.

Q. Find the total surface area of the triangular prism.



A. $P_b = 6 + 5 + 5 = 16$

$A_b = \frac{1}{2} bh$ 

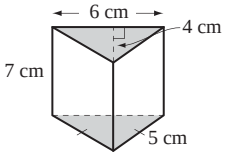
$= \frac{1}{2} \times (6 \times 4) = 12$

$TSA = P_b h + 2A_b$

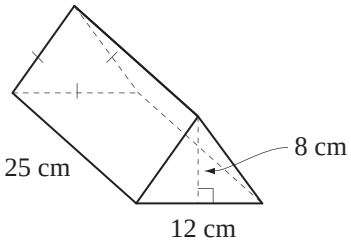
$= 16 \times 7 + 2 \times 12$

$= 112 + 24$

$= \mathbf{136\ cm^2}$



a) Find the total surface area of the triangular prism.



First find the perimeter and area of the base

$P_b = 12 + 12 + 12 = 36$

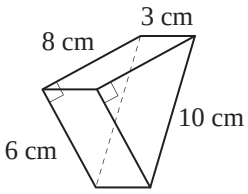
$A_b = \frac{1}{2} \times (12 \times 8) = 48$

$TSA = P_b h + 2A_b$

$= 36 \times 25 + 2 \times 48$

$= 900 + 96$ = ____ cm^2

b) Find the total surface area of the triangular prism.



$P_b =$

$A_b =$

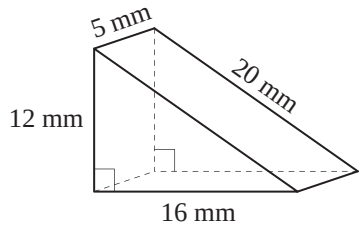
$TSA =$

$=$

$=$ ____ $=$ ____ cm^2

Skill 24.4 Calculating the total surface area (*TSA*) of triangular prisms (2).

c) Find the total surface area of the triangular prism.



P_b =

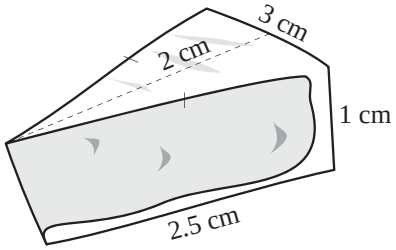
A_b =

$TSA = P_b h + 2A_b$

=

= = mm^2

d) Find the total surface area of the triangular prism shaped slice of cheese.



P_b =

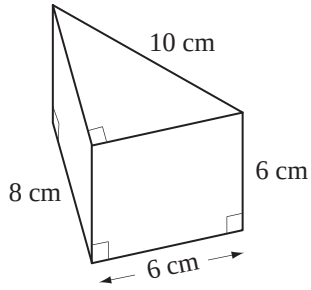
A_b =

$TSA = P_b h + 2A_b$

=

= = cm^2

e) Find the total surface area of the triangular prism.



P_b =

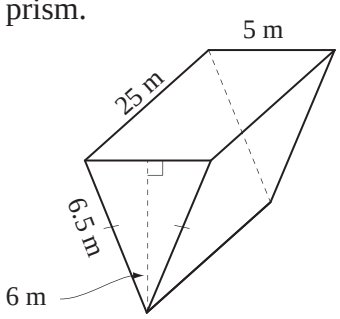
A_b =

TSA =

=

= = cm^2

f) Find the total surface area of the triangular prism.



P_b =

A_b =

TSA =

=

= = m^2

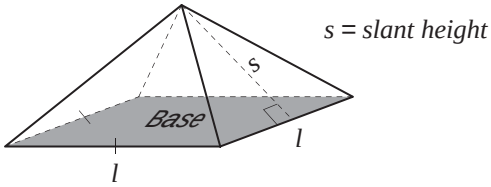
Skill 24.5 Calculating the total surface area (*TSA*) of pyramids (1).

- Find any unknown side lengths.
- Calculate the area of each face.
- Add together the area of all faces.

OR

- Substitute values into the formulas:

regular square pyramid

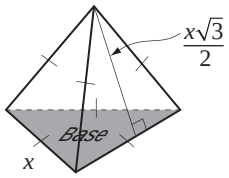


TSA = Area of square base + 4 × Area of triangle

$TSA = A_b + 4 \times \frac{1}{2} ls$

$TSA = l^2 + 2ls$

regular triangular pyramid (regular tetrahedron)



TSA = 4 × Area of equilateral triangle

$TSA = 4 \times \frac{1}{2} x \times \frac{x\sqrt{3}}{2}$

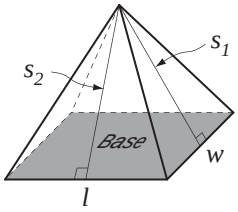
$TSA = x^2\sqrt{3}$

rectangular pyramid

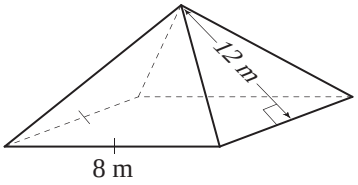
TSA = Area of base + 2 × Area of triangles left & right + 2 × Area of triangles front & back

$TSA = B + 2 \times \frac{1}{2} ws_1 + 2 \times \frac{1}{2} ls_2$

$TSA = lw + ws_1 + ls_2$



Q. Find the total surface area of the regular square pyramid.



A. *TSA* = $l^2 + 2ls$ *where* $l = 8$ *and* $s = 12$

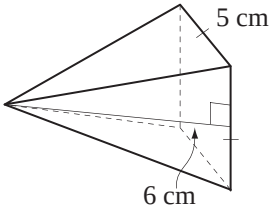
$= 8 \times 8 + 2 \times 8 \times 12$

$= 64 + 16 \times 12$

$= 64 + 192$

$= \mathbf{256\ m^2}$

a) Find the total surface area of the regular square pyramid.



TSA = $l^2 + 2ls$ *where* $l = 5$ *and* $s = 6$

$= 5 \times 5 + 2 \times 5 \times 6$

$= 25 + 60$ = ____ cm^2

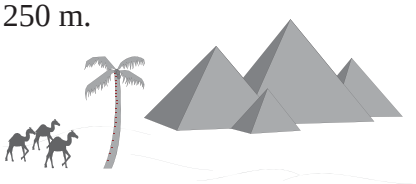
b) Find the total surface area of one of the salt and pepper shakers given that they are regular square pyramids of base side length 3 cm and slant height 4 cm.

TSA = $l^2 + 2ls$

= ____ cm^2

Skill 24.5 Calculating the total surface area (*TSA*) of pyramids (2).

c) Find the total surface area of the largest regular square pyramid below. It has a base side length of 200 m and slant height of 250 m.



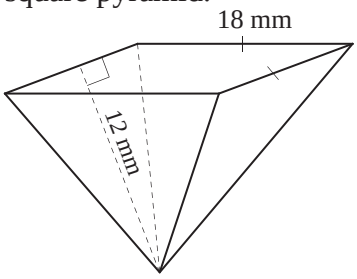
TSA =

.....

.....

= m^2

d) Find the total surface area of the regular square pyramid.



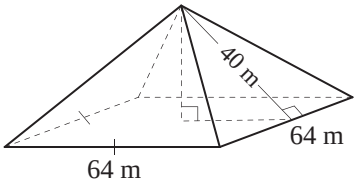
TSA =

.....

.....

= mm^2

e) Find the surface area of the regular square pyramid.



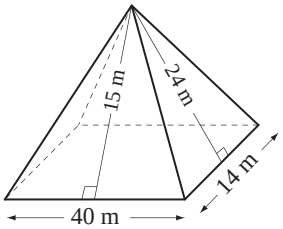
TSA =

.....

.....

= m^2

f) Find the surface area of the rectangular pyramid.



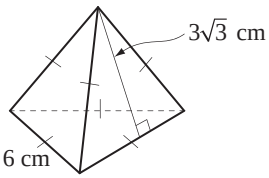
TSA =

.....

.....

= m^2

g) Find the surface area of the regular tetrahedron. [Give your answer as a surd.]



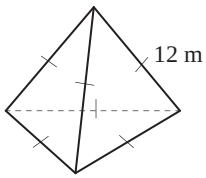
TSA =

.....

.....

= cm^2

h) Find the surface area of the regular tetrahedron. [Give your answer as a surd.]



TSA =

.....

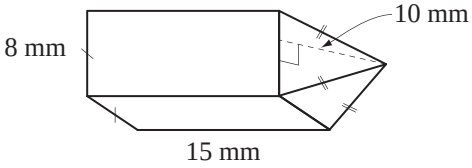
.....

= m^2

Skill 24.6 Calculating the total surface area of composite solids (1).

- Break the solid into workable parts.
- Calculate the total surface area of each solid. (see skills 24.2, page 279 and 24.3, page 280)
- Add the results.

Q. Find the total surface area of the obelisk.



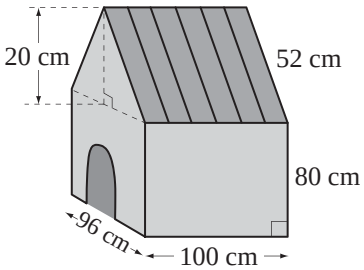
l = 8 (length)
s = 10 (slant height)

A. *TSA regular square pyramid (without base)*
= 2ls where l = 8 and s = 10
= 2 × 8 × 10
= 160

TSA square prism (without base)
= 4lh + l^2 where l = 8 and h = 15
= 4 × (8 × 15) + 8 × 8
= 4 × 120 + 64 = 544

TSA obelisk = 160 + 544 = ***704 mm²***

a) Disregarding the entrance, find the total surface area of the doghouse, excluding its floor.



TSA roof prism = 2 × 100 × 52 + 2 × $\frac{1}{2}$ × 96 × 20 = ...

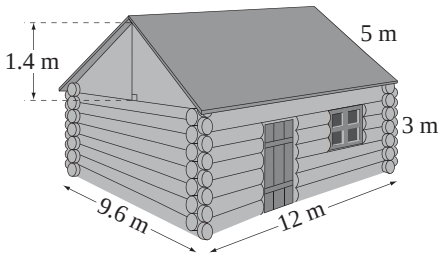
= .....

TSA base prism = 2 × 100 × 80 + 2 × 96 × 80 = ...

= .....

TSA house = ... = ... cm^2

b) Disregarding the door and windows, find the total surface area of the log cabin, excluding its floor.



TSA roof prism = ...

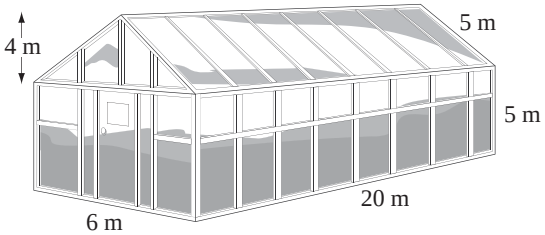
= .....

TSA base prism = ...

= .....

TSA cabin = ... = ... m^2

c) Find the total surface area of the glass house, excluding its floor.



TSA roof prism = ...

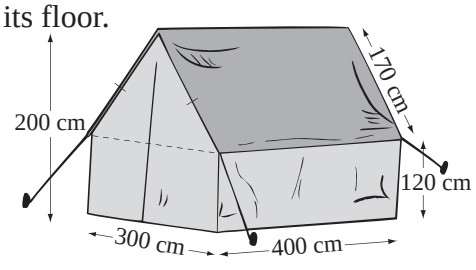
= .....

TSA base prism = ...

= .....

TSA house = ... = ... m^2

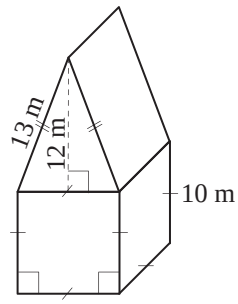
d) Find the total surface area of the tent canvas excluding its floor.



..... = ... cm^2

Skill 24.6 Calculating the surface area of composite solids (2).

e) Find the total surface area of the solid.



Roof

$P_b = 36$ $A_b = \frac{1}{2} \times 10 \times 12 = 60$

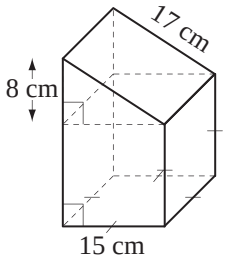
TSA prism =

TSA prism – face =

TSA cube – face =

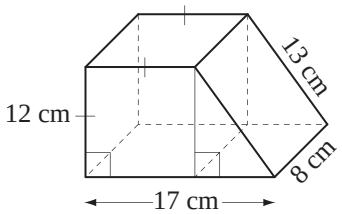
TSA solid = = m²

f) Find the total surface area of the solid.



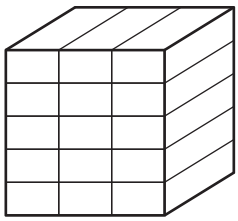
TSA = = cm²

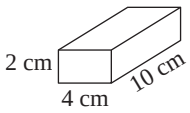
g) Find the total surface area of the solid.



TSA = = cm²

h) Bernie bought a rectangular box containing 15 tightly packaged erasers. What is the total surface area of the box?

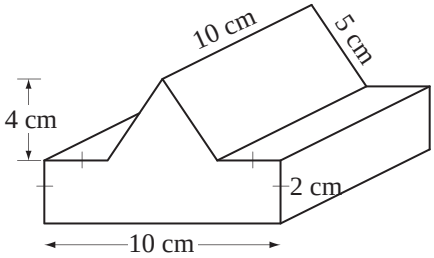




TSA = = cm²

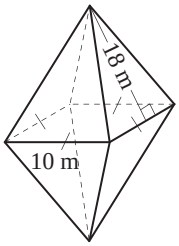
Skill 24.6 Calculating the surface area of composite solids (3).

i) Find the total surface area of the prism.



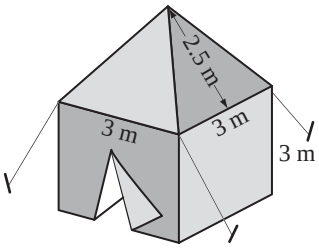
TSA = = cm^2

j) Find the total surface area of the octahedron.



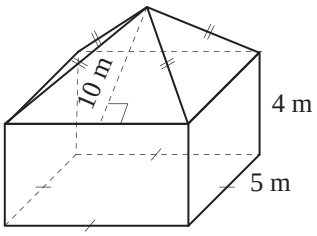
TSA = = m^2

k) Disregarding the entrance, find the total surface area of the marquee canvas excluding its floor.



TSA = = m^2

l) Find the total surface area of the obelisk.



TSA = = m^2

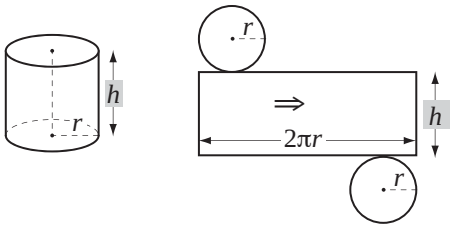
Skill 24.7 Calculating the total surface area (*TSA*) of basic 3-dimensional round shapes (1).

- Substitute values into the formulas:

cylinder

$TSA = 2\pi r^2 + 2\pi rh$

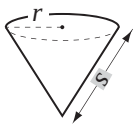
$TSA = 2\pi r(r + h)$



cone

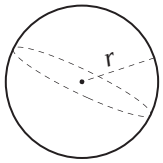
$TSA = \pi r^2 + \pi rs$

$TSA = \pi r(r + s)$

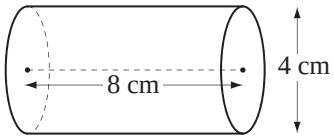


sphere

$TSA = 4\pi r^2$

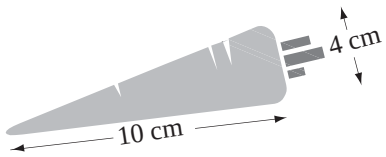


Q. Using $TSA = 2\pi r(r + h)$ and $\pi \approx 3.14$, find the total surface area of the cyclinder.



A. *TSA = 2πr(r + h) where r = 2 and h = 8*
= 2 × 3.14 × 2 × (2 + 8)
= 12.56 × 10
= 125.6 cm²

a) Use $TSA = \pi r(r + s)$ and $\pi \approx 3.14$, to find the total surface area of the conical carrot.

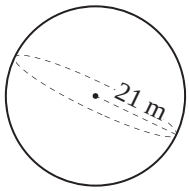


TSA = πr(r + s) where r = 2 and s = 10

= 3.14 × 2 × (2 + 10)

= 6.28 × 12 = 75.36 cm²

b) Using $TSA = 4\pi r^2$ and $\pi \approx \frac{22}{7}$, find the total surface area of the sphere.

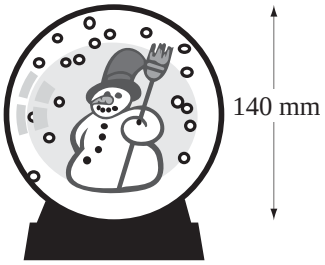


TSA =

= ..... = ... m²

Skill 24.7 Calculating the total surface area (*TSA*) of basic 3-dimensional round shapes (2).

c) Using $TSA = 4\pi r^2$ and $\pi \approx \frac{22}{7}$, find the total surface area of the snow globe.

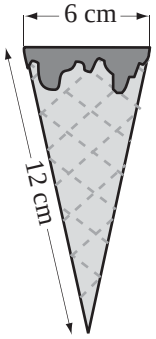


TSA = ...

= .....

= ..... = mm²

d) Use $TSA = \pi r(r + s)$ and $\pi \approx 3.14$ to find how much area still needs to be covered in chocolate to cover the whole cone only on the outside, given that 40 cm² have been covered so far.

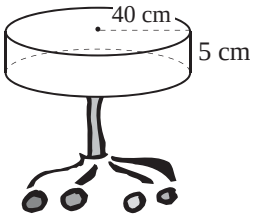


TSA = ...

= .....

= ..... = cm²

e) Using $TSA = 2\pi r(r + h)$ and $\pi \approx 3.14$, find the total surface area of the cyclindrical stool seat.

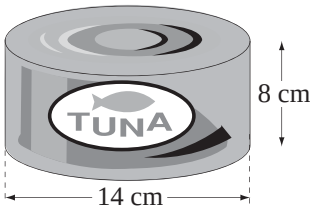


TSA = ...

= .....

= ..... = cm²

f) Using $TSA = 2\pi r(r + h)$ and $\pi \approx \frac{22}{7}$, find the total surface area of the can of tuna.



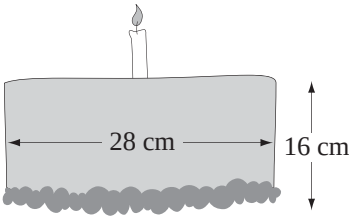
TSA = ...

= .....

= ..... = cm²

g) Using *TSA* of a cylinder = $2\pi r(r + h)$ and $\pi \approx \frac{22}{7}$, find the total surface area of the icing.

[N.B. The base of the cake is not iced.]



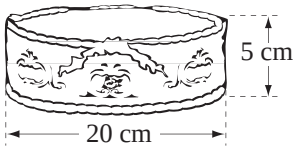
TSA = ...

= .....

= .....

= ..... = cm²

h) This wedding cake is covered in white icing, except for the base. Using $\pi \approx 3.14$ find the total surface area of the white icing.



TSA = ...

= .....

= .....

= ..... = cm²

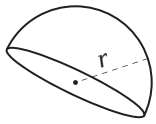
Skill 24.8 Calculating the total surface area (*TSA*) of more complex 3-dimensional round shapes.

- Substitute values into the appropriate formula.
- Adapt formulas where necessary.

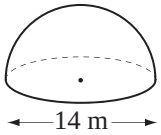
hemisphere

$$TSA = \frac{4\pi r^2}{2} + \pi r^2$$

$$TSA = 3\pi r^2$$



Q. Using $\pi \approx \frac{22}{7}$ find the total surface area of the hemisphere.



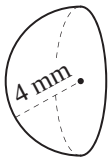
A. *TSA = 3πr² where r = 7 m*

$= 3 \times \frac{22}{1\cancel{7}} \times \cancel{7}^{1} \times 7$

$= 66 \times 7$

$= \mathbf{462\ m^2}$

a) Using the total surface area of a sphere $TSA = 4\pi r^2$ and $\pi \approx 3.14$, find the total surface area of the hemisphere.

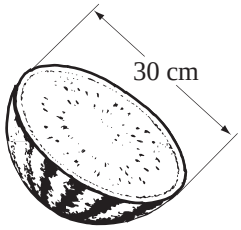


TSA = 3πr²

= 3 × 3.14 × 4 × 4

= 9.42 × 16 = ____ mm²

b) The total surface area of a sphere is $TSA = 4\pi r^2$. Using $\pi \approx 3.14$ find the total surface area of the watermelon half.

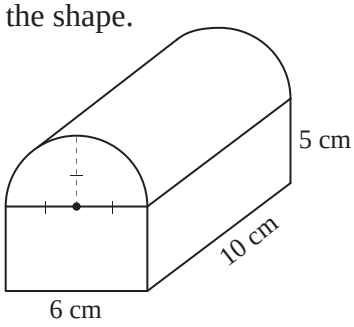


TSA =

=

= ____ = ____ cm²

c) Use $\pi \approx 3.14$ to find the total surface area of the shape.



TSA prism =

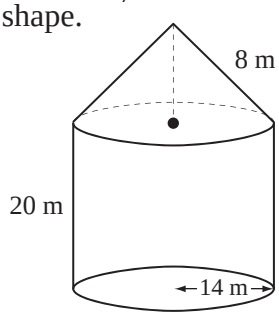
=

TSA cylinder half =

=

TSA = ____ = ____ cm²

d) Use $\pi \approx \frac{22}{7}$ to find the total surface area of the shape.



LA cone =

=

TSA cylinder =

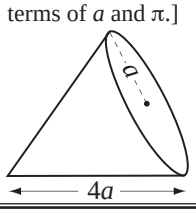
=

TSA = ____ = ____ m²

Skill 24.9 Expressing the total surface area (*TSA*) of 3-dimensional shapes in algebraic form.

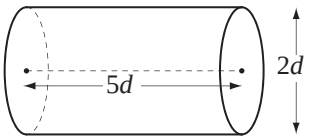
- Substitute values into the appropriate formula for total surface area.
  (see skills 24.2 to 24.5, pages 279 to 284, skills 24.7, page 289 and 27.8, page 291)
- Adapt formulas where necessary.

Q. Write an algebraic expression for the total surface area *TSA* of the cone. [Express the answer in terms of a and π.]



A. $TSA = \pi r(r + s)$ *where* $r = a$ *and* $s = 4a$
$= \pi \times a \times (a + 4a)$
$= \pi \times a \times 5a$
$= \mathbf{5\pi a^2}$

a) Write an algebraic expression for the total surface area *TSA* of the cylinder. [Express the answer in terms of d and π.]



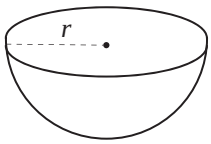
$TSA = 2\pi r(r + h)$ *where* $r = d$ *and* $h = 5d$

$= 2\pi d(d + 5d)$

$= 2\pi d \times 6d$

$TSA = 12\pi d^2$

b) Write an algebraic expression for the total surface area *TSA* of the hemisphere. [Express the answer in terms of r and π.]



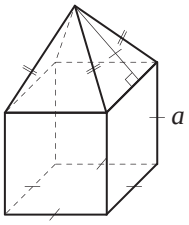
$TSA =$

$=$

$=$

$TSA =$

c) Write an algebraic expression for the total surface area *TSA* of the obelisk. [Express the answer in terms of a.]



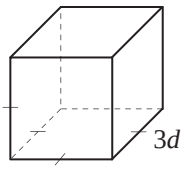
$TSA =$

$=$

$=$

$TSA =$

d) Write an algebraic expression for the total surface area *TSA* of the cube. [Express the answer in terms of d.]



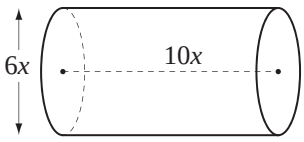
$TSA =$

$=$

$=$

$TSA =$

e) Write an algebraic expression for the total surface area *TSA* of the cylinder. [Express the answer in terms of x and π.]



$TSA =$

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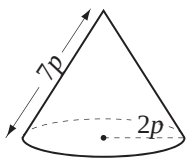
f) Write an algebraic expression for the total surface area *TSA* of the cone. [Express the answer in terms of p and π.]



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