25. [Volume]

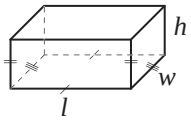
Skill 25.1 Calculating the volume of square and rectangular prisms.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

rectangular prism

V = length × width × height

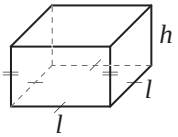
$V = lwh$



square prism

V = length × width × height

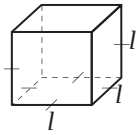
$V = l^2h$



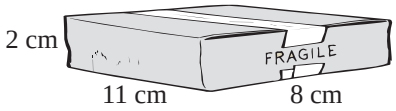
cube

V = length × length × length

$V = l^3$



Q. The parcel is a rectangular prism. What is the volume of the parcel?



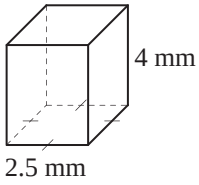
A. $V = lwh$ *where* $l = 11$, $w = 8$ *and* $h = 2$

$= 11 \times 8 \times 2$

$= 88 \times 2$

$= \mathbf{176\ cm^3}$

a) Find the volume of the square prism.

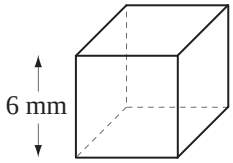


$V = l^2h$ *where* $l = 2.5$ *and* $h = 4$

$= 2.5 \times 2.5 \times 4$

$= 2.5 \times 10$ = ____ mm^3

b) Find the volume of the cube.

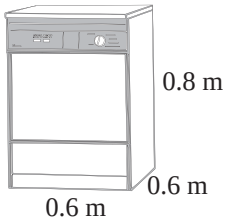


$V = l^3$

=

= = ____ mm^3

c) Given that the dishwasher is a square prism, find its volume.

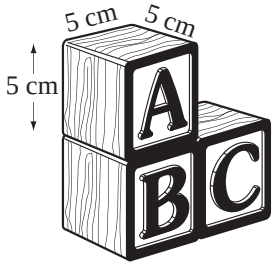


V =

=

= = ____ m^3

d) Find the volume of the building blocks stack.



V =

=

= = ____ cm^3

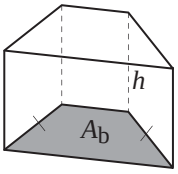
Skill 25.2 Calculating the volume of other prisms (1).

- Find any unknown side lengths.
- Calculate the area of the base. Use known formulae where possible.
  Hint: The height of a triangle is needed to calculate the area of a triangle and should not be confused with the height (h) of the prism.
- Substitute known values into the formula:

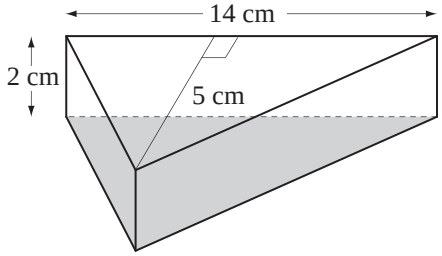
prism

V = Area of base × height of prism

$V = A_b h$



Q. Find the volume of the triangular prism using $V = A_b h$.



A. $V = A_b h$ — *h = height of prism*

$A_b = \frac{1}{2} bh$ — *h = height of triangle*

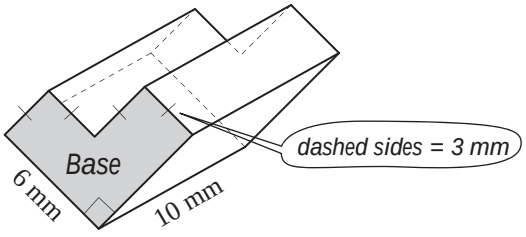
$= \frac{1}{2} \times 14^{7} \times 5$ — *First calculate area of base*

$= 7 \times 5 = 35$

$V = 35 \times 2$

$= \mathbf{70\ cm^3}$

a) Find the volume of the prism.

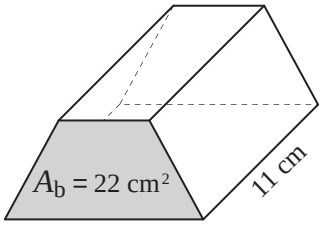


$V = A_b h$ *where* $h = 10$ *mm*

$A_b = 3 \times 3 + 6 \times 3 = 9 + 18 = 27$

$V = 27 \times 10$ = ____ mm³

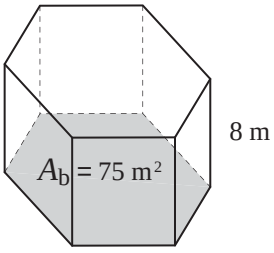
b) Using V = Area of base (A_b) × height (h), find the volume of the prism.



$V = A_b h$

= ____ = ____ cm³

c) Using $V = A_b h$ find the volume of the hexagonal prism.

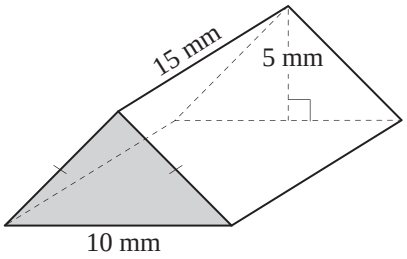


$V = A_b h$

= ____

= ____ = ____ m³

d) Find the volume of the triangular prism using $V = A_b h$.



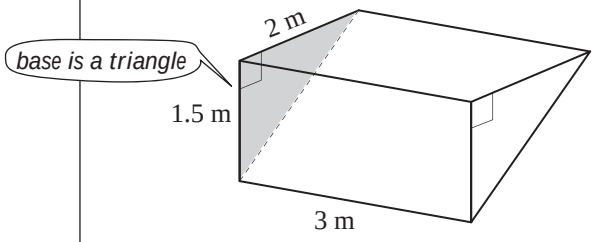
$V = A_b h$

A_b = ____

V = ____ = ____ mm³

Skill 25.2 Calculating the volume of other prisms (2).

e) Find the volume of the prism.

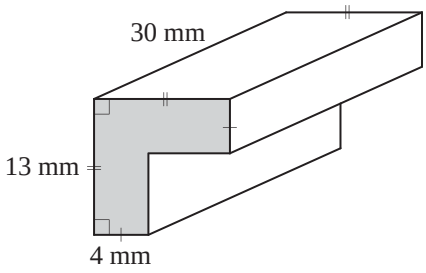


$V = A_b h$ *First calculate area of base*

$A_b = \frac{1}{2} \times 1.5 \times 2 = 1.5$

$V = 1.5 \times 3$ = ____ m^3

f) Find the volume of the prism.

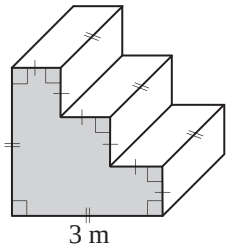


V =

A_b =

V = = ____ mm^3

g) Find the volume of concrete used to build the steps.

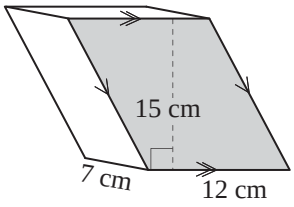


V =

A_b =

V = = ____ m^3

h) Find the volume of the prism.

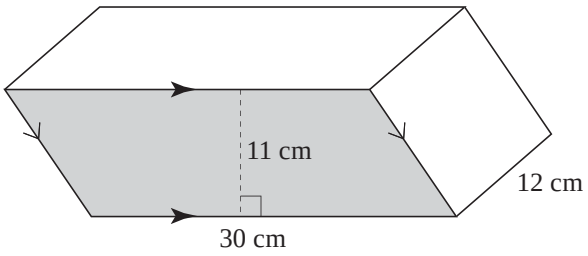


V =

A_b =

V = = ____ cm^3

i) Find the volume of the prism.

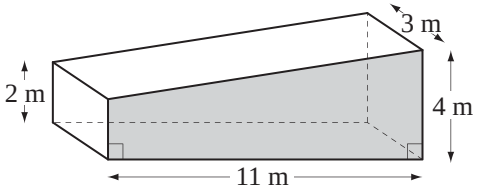


V =

A_b =

V = = ____ cm^3

j) Find the volume of the prism.



V =

A_b =

V = = ____ m^3

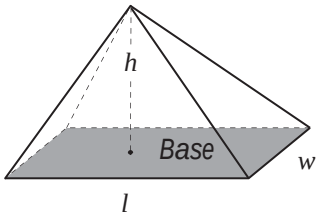
Skill 25.3 Calculating the volume of pyramids.

- Substitute known values into the appropriate formula to find the area of the base.
- Substitute known values into the volume formula:

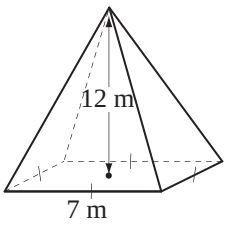
pyramid

$V = \frac{1}{3} \times$ Area of base $\times$ height of pyramid

$V = \frac{A_b h}{3}$



Q. Find the volume of the square pyramid.



A. $V = \frac{A_b h}{3}$

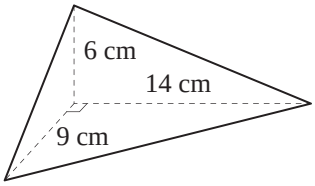
$A_b = 7 \times 7 = 49$

$V = \frac{49 \times 12^{4}}{\cancel{3}}$ *Simplify: ÷ 3*

$= 49 \times 4$

$= \mathbf{196\ m^3}$

a) Using $V = \frac{A_b h}{3}$ find the volume of the triangular pyramid of height 6 cm.

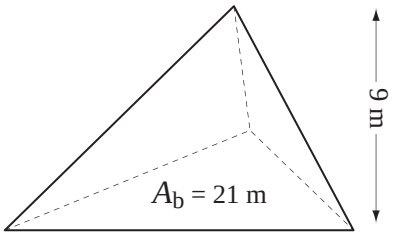


$V = \frac{A_b h}{3}$

$A_b = \frac{1}{2} bh = \frac{1}{\cancel{2}} \times 9 \times \cancel{14}^{7} = 63$

$V = \frac{63 \times \cancel{6}^{2}}{\cancel{3}}$ = ____ cm^3

b) Using $V = \frac{A_b h}{3}$ find the volume of the triangular pyramid.

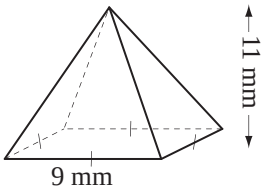


$V = \frac{A_b h}{3}$

$A_b =$

$V =$ = ____ m^3

c) Find the volume of the square pyramid.

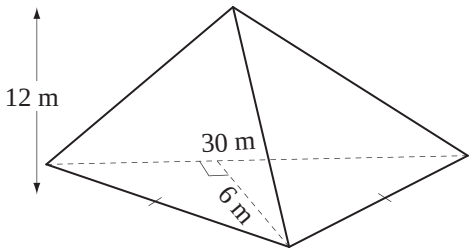


$V =$

$A_b =$

$V =$ = ____ mm^3

d) Find the volume of the triangular pyramid.



$V =$

$A_b =$

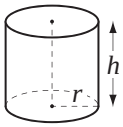
$V =$ = ____ m^3

Skill 25.4 Calculating the volume of basic 3-dimensional round shapes.

- Substitute known values into the appropriate formula:

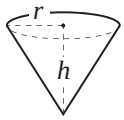
cylinder

$V = \pi r^2 h$



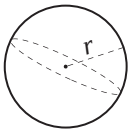
cone

$V = \dfrac{\pi r^2 h}{3}$

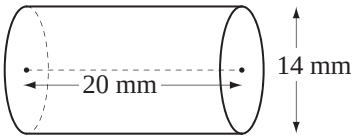


sphere

$V = \dfrac{4\pi r^3}{3}$



Q. Using $V = \pi r^2 h$ and $\pi \approx \dfrac{22}{7}$, find the volume of the cylinder.



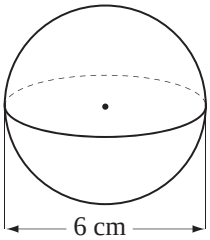
A. $V = \pi r^2 h$ *where* $r = 7$ *and* $h = 20$

$= \dfrac{22}{\cancel{7}} \times \cancel{7} \times 7 \times 20$ *Simplify: ÷ 7*

$= 154 \times 20$

$= \boldsymbol{3080\ mm^3}$

a) Using $V = \dfrac{4\pi r^3}{3}$ and $\pi \approx 3.14$, find the volume of the sphere.

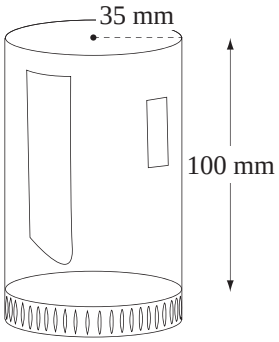


$V = \dfrac{4\pi r^3}{3}$ *where* $r = 3$ *cm*

$= \dfrac{4 \times 3.14 \times \cancel{3} \times 3 \times 3}{\cancel{3}}$ *Simplify: ÷ 3*

$= 36 \times 3.14$ = ____ cm^3

b) Using $V = \pi r^2 h$ and $\pi \approx \dfrac{22}{7}$, find the maximum volume of the glass.

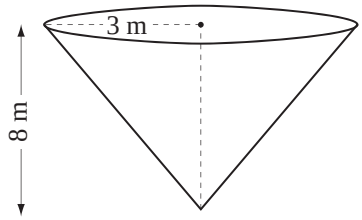


$V =$

$=$

$=$ ____ = ____ mm^3

c) Using $V = \dfrac{\pi r^2 h}{3}$ and $\pi \approx 3.14$, find the volume of the cone.

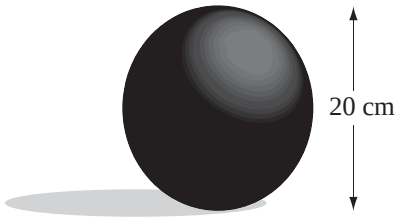


$V =$

$=$

$=$ ____ = ____ m^3

d) Using $V = \dfrac{4\pi r^3}{3}$ and $\pi \approx 3.14$, find the volume of the sphere, correct to 2 decimal places.



$V =$

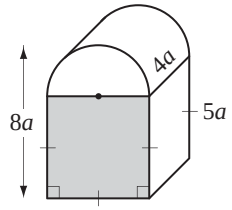
$=$

$=$ ____ = ____ cm^3

Skill 25.5 Expressing the volume of 3-dimensional shapes in algebraic form.

- Substitute values into the appropriate formula for volume. (see skills 25.1 to 25.4, pg. 293 to 297)
- Adapt the formulas where necessary.

Q. Write an algebraic expression for the volume V of the shape. [Express the answer in terms of a and π.]



A. $V_{sq.\ prism} = l^2h$ *where* $l = 5a$ *and* $h = 4a$

$= 5a \times 5a \times 4a = 100a^3$

$V_{half\ cyl.} = \frac{1}{2}\pi r^2 h$ *where* $r = 3a$ *and* $h = 4a$

$= \frac{1}{2} \pi \times 9a^2 \times 4a = 18\pi a^3$

$V_{shape} = 100a^3 + 18\pi a^3 = \mathbf{2a^3(50 + 9\pi)}$

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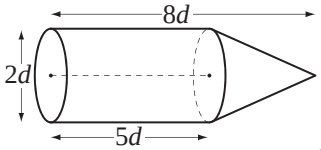
a) Write an algebraic expression for the volume V of the shape. [Express the answer in terms of d and π.]



r = d and h = 5d

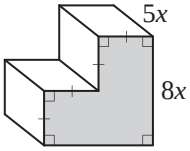
$V_{cyl} = \pi r^2 h = \pi \times d^2 \times 5d = 5\pi d^3$

r = d and h = 3d

$V_{cone} = \frac{\pi r^2 h}{3} = \frac{1}{3} \times \pi \times d^2 \times 3d = \pi d^3$

$V_{shape} = 5\pi d^3 + \pi d^3$ = $V =$

b) Write an algebraic expression for the volume V of the prism. [Express the answer in terms of x.]



$V =$

$V =$

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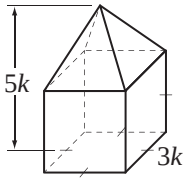
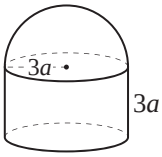
c) Write an algebraic expression for the volume V of the obelisk. [Express the answer in terms of k.]



$V =$

$V =$

d) Write an algebraic expression for the volume V of the shape. [Express the answer in terms of a and π.]



$V =$

$V =$

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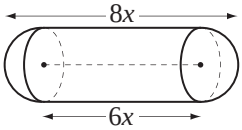
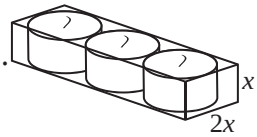
e) Write an algebraic expression for the volume V of the capsule. [Express the answer in terms of x and π.]



$V =$

$V =$

f) A rectangular box contains 3 identical candles placed with no room to move. Write an algebraic expression in terms of x and π for the volume of the box which is **not** occupied by the candles.



$V =$

$V =$

Skill 25.6 Calculating volume in relation to capacity.

- Substitute known values into the appropriate formula.
- Use the conversion factors between cubic units and capacity units:

Conversion Facts - CUBIC VOLUME to CAPACITY

$1000\ cm^3 = 1\,000\ mL = 1\ L$

$1000\ L = 1\ m^3$

Q. A rectangular swimming pool is 20 m long and 12 m wide. If its average depth is 2 m, how many litres of water would you need to fill the pool? [Hint: $1000\ L = 1\ m^3$]

A. *$V = lwh$ where $l = 20$, $w = 12$ and $h = 2$*

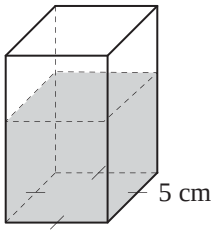
$= 20 \times 12 \times 2$

$= 20 \times 24$

$= 480\ m^3$ (Convert m^3 to L)

$= 480\,000\ L$

a) The vase has 0.5 litre of water in it. Find the depth of the water. [Hint: $1000\ cm^3 = 1\ L$]



(Using 0.5 L = 500 mL)

$V = l^2h$ where $l = 5$ and $V = 500$

$500 = 5 \times 5 \times h \Rightarrow 25h = 500$

$25h \div 25 = 500 \div 25$

$h = 20$ cm

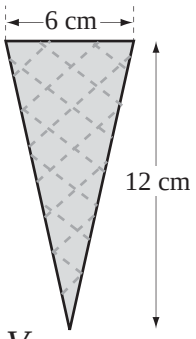
b) A rectangular fish tank with dimensions 20 cm by 15 cm by 10 cm is half full of water. How many millilitres of water would you need to fill the fish tank? [Hint: $1\ mL = 1\ cm^3$]

$=$

$=$

$=$ mL

c) Using $V = \frac{\pi r^2 h}{3}$ and $\pi \approx 3.14$, find how much ice cream could fit exactly inside this cone.
[Hint: $1\ mL = 1\ cm^3$]

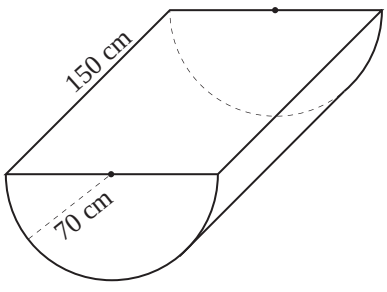


$V =$

$=$

$=$ $=$ mL

d) Using $\pi \approx \frac{22}{7}$ find the maximum volume of water the trough could hold.
[Hint: $1000\ cm^3 = 1\ L$]



$V =$

$=$

$=$ $=$ L

Skill 25.7 Calculating volume in relation to length and area.

- Substitute known values into the appropriate formulas for area and volume.

Q. A rectangular prism with volume 216 cm^3 has a height of 6 cm and a width of 5 cm. Calculate the length of the prism.

A. *V = lwh where V = 216, w = 5 and h = 6*
216 = l × 5 × 6
30l = 216
l = 216 ÷ 30 (Divide 21.6 by 3)
l = ***7.2 cm***

a) A cube has a total surface area of 54 cm^2. What is the volume of the cube?

TSA = 6l^2 and V = l^3 (In a cube: l = w = h)

54 = 6l^2 so l^2 = $\frac{54}{6}$ = 9 and l = 3

V = 3^3 = 27 cm^3

b) A rectangular prism with volume 189 mm^3 has a height of 3 mm and a length of 7 mm. Calculate the width of the prism.

mm

c) If a cube has a total surface area of 96 mm^2, what is the volume of the cube?

mm^3

d) If a cube has a total surface area of 150 cm^2, what is the volume of the cube?

cm^3

e) A rectangular long jump pit holds 13.5 m^3 of sand. If the pit is 9 m long and 3 m wide, how deep is the sand?

m

f) How many metal cubes of side length 4 mm need to be melted down to produce a single cube of side length 8 mm?

g) A rectangular fish tank can hold 30 000 cm^3 when full. If the tank is 20 cm wide and 30 cm long, how deep is the water?

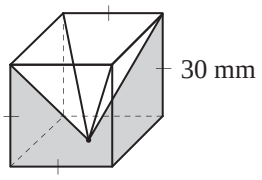
cm

h) How many metal cubes of side length 3 cm need to be melted down to produce a single cube of side length 9 cm?

Skill 25.8 Calculating the volume of composite solids.

- Substitute values into the appropriate formulas for volume.

Q. A 30 mm × 30 mm × 30 mm pyramid is removed from a 30 mm × 30 mm × 30 mm cube. Find the volume of the remaining shape.



A. V_1 *of cube* $= l^3$

V_2 *of square pyramid* $= \frac{A_b h}{3}$ *where* $A_b = l^2$ *and* $h = l$

$= \frac{l^3}{3}$

$V_1 - V_2 = l^3 - \frac{l^3}{3}$

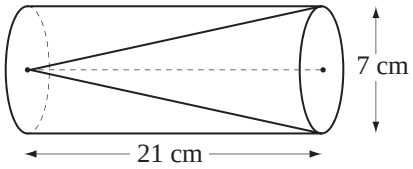
$V = \frac{2l^3}{3}$ *where* $l = 30$

$V = \frac{2 \times \overset{10}{\cancel{30}} \times 30 \times 30}{\underset{1}{\cancel{3}}}$ *Simplify: ÷ 3*

$V = 20 \times 900$

$= \mathbf{18\,000\ mm^3}$

a) How much less is the volume of the cone than the volume of the cylinder of the same height? (Use $\pi \approx \frac{22}{7}$)



V_1 *of a cylinder* $= \pi r^2 h$, V_2 *of a cone* $= \frac{\pi r^2 h}{3}$

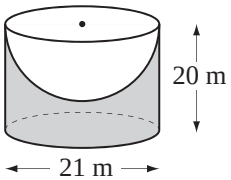
$V_1 - V_2 = \pi r^2 h - \frac{\pi r^2 h}{3}$

$= \frac{2\pi r^2 h}{3}$

$= \cancel{2} \times \frac{\cancel{22}^{11}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{\cancel{2}} \times \cancel{21}^{7} \times \frac{1}{\cancel{3}}$ *Simplify*

$= 11 \times 7 \times 7$ = cm³

b) A hemisphere of diameter 21 m is removed from this cylinder. Using $\pi \approx \frac{22}{7}$ find the volume of the remaining shape.



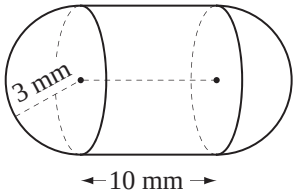
$V =$

=

=

= = m³

c) Using $\pi \approx 3.14$ find the volume of the shape.

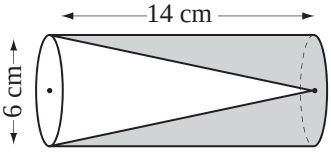


$V =$

=

= = mm³

d) A cone of diameter 6 cm and height 14 cm is removed from this cylinder. Find the volume of the remaining shape. (Use $\pi \approx \frac{22}{7}$)



$V =$

=

= = cm³