

# 13. [Operations]

Skill 13.1 Using the commutative property for addition.

MM3.2 1 1 2 2 3 3 4 4  
MM4.1 1 1 2 2 3 3 4 4

## COMMUTATIVE PROPERTY for +

$$\begin{array}{r} 2 + 5 = 7 \\ 5 + 2 = 7 \end{array}$$

You can add numbers in any order and not change the outcome.

$$\text{SO } 2 + 5 = 5 + 2$$

Q.  $6 + 3 = 3 + 6$

True or false?

A. **true**

Solve both sides of the equation and compare the results.

$$6 + 3 = 9$$

$$3 + 6 = 9$$

The results are the same.

a)  $10 - 4 = 4 - 10$

True or false?

b)  $4 + 5 = 5 + 4$

True or false?

c)  $7 + 9 = 9 + 7$

True or false?

$$10 - 4 = 6 \text{ but}$$

$$4 - 10 \neq 6$$

false

d)  $9 - 3 = 3 - 9$

True or false?

e)  $2 + 9 = 9 + 2$

True or false?

f)  $8 - 1 = 1 - 8$

True or false?

g)  $\boxed{8} + 2 = 2 + 8$

h)  $9 + 6 = \boxed{\phantom{0}} + 9$

i)  $4 + 1 = \boxed{\phantom{0}} + 4$

j)  $\boxed{\phantom{0}} + 5 = 5 + 2$

k)  $\boxed{\phantom{0}} + 7 = 7 + 5$

l)  $3 + 9 = \boxed{\phantom{0}} + 3$

m)  $\boxed{\phantom{0}} + 13 = 13 + 6$

n)  $17 + 10 = \boxed{\phantom{0}} + 17$

o)  $11 + 19 = \boxed{\phantom{0}} + 11$

p)  $\boxed{\phantom{0}} + 22 = 22 + 14$

q)  $\boxed{\phantom{0}} + 17 = 17 + 12$

r)  $15 + 18 = \boxed{\phantom{0}} + 15$

s)  $13 + \boxed{\phantom{0}} = 31 + 13$

t)  $\boxed{\phantom{0}} + 16 = 16 + 28$

u)  $27 + \boxed{\phantom{0}} = 7 + 27$

**COMMUTATIVE PROPERTY for ×**

$2 \times 5 = 10$

$5 \times 2 = 10$

You can multiply numbers in any order  
and not change the outcome.

SO  $2 \times 5 = 5 \times 2$

Q.  $\boxed{\quad} \times 5 = 5 \times 9$

**A. 9**

Ask: "What number multiplied by 5 equals 5 multiplied by 9?"

Answer:  $9 \times 5 = 5 \times 9$ 

**a)**  $10 \div 2 = 2 \div 10$

True or false?

**b)**  $4 \times 5 = 5 \times 4$

True or false?

**c)**  $7 \times 9 = 9 \times 7$

True or false?

$10 \div 2 = 5$  but

$2 \div 10 \neq 5$

**false**

**d)**  $9 \div 3 = 3 \div 9$

True or false?

**e)**  $6 \times 7 = 7 \times 6$

True or false?

**f)**  $12 \div 4 = 4 \div 12$

True or false?

**g)**  $8 \times 2 = 2 \times 8$

**h)**  $\boxed{\quad} \times 5 = 5 \times 2$

**i)**  $4 \times 1 = \boxed{\quad} \times 4$

**j)**  $9 \times 6 = \boxed{\quad} \times 9$

**k)**  $\boxed{\quad} \times 4 = 4 \times 12$

**l)**  $19 \times 10 = \boxed{\quad} \times 19$

**m)**  $11 \times 3 = \boxed{\quad} \times 11$

**n)**  $\boxed{\quad} \times 6 = 6 \times 18$

**o)**  $\boxed{\quad} \times 13 = 13 \times 12$

**p)**  $7 \times 18 = \boxed{\quad} \times 7$

**q)**  $\boxed{\quad} \times 24 = 24 \times 17$

**r)**  $13 \times 15 = \boxed{\quad} \times 13$

**s)**  $\boxed{\quad} \times 5 = 5 \times 4$

**t)**  $11 \times \boxed{\quad} = 10 \times 11$

**u)**  $\boxed{\quad} \times 16 = 16 \times 7$

**IDENTITY ELEMENT for + is ZERO**

$14 + 0 = 14$

The sum of zero and any number is that number.

Q.  + 0 = 2

A. 2 Ask: "What number added to zero makes 2?"

Answer:  $2 + 0 = 2$ 

a)  $10 + 0 = 10$

True or false?

 true

b)  $6 + 0 = 0$

True or false?

c)  $0 + 7 = 7$

True or false?

d)  $0 + 8 = 8$

True or false?

e)  $3 - 0 = 0$

True or false?

f)  $9 - 0 = 9$

True or false?

g)  8 + 0 = 8

h)  + 0 = 5

i)  $3 + 0 =$

j)  $9 +$  = 9

k)  $2 -$  = 0

l)  $5 -$  = 5

m) Which expression equals 7?

- A)  $0 + 7$
- B)  $0 \times 7$
- C)  $0 - 7$

n) Which expression equals 8?

- A)  $0 \times 8$
- B)  $0 - 8$
- C)  $0 + 8$

o) Which expression equals 5?

- A)  $1 + 5$
- B)  $1 \times 5$
- C)  $1 - 5$

p) Which expression equals 3?

- A)  $3 + 0$
- B)  $0 - 3$
- C)  $3 \times 0$

q) Which expression equals 4?

- A)  $1 + 4$
- B)  $1 - 4$
- C)  $1 \times 4$

r) Which expression equals 6?

- A)  $6 \times 0$
- B)  $0 - 6$
- C)  $6 + 0$

**IDENTITY ELEMENT for  $\times$  is ONE**

$14 \times 1 = 14$

The product of one and any number is that number.

Q.  $\boxed{\phantom{00}} \times 1 = 8$

A. **8**

Ask: "What number multiplied by 1 makes 8?"

Answer:  $8 \times 1 = 8$

Q. Which expression equals 13?

- A)  $1 + 13$
- B)  $1 \times 13$
- C)  $1 \div 13$

A. **B**

Solve all expressions and then compare the results.

- A)  $1 + 13 \neq 13$
- B)  $1 \times 13 = 13$
- C)  $1 \div 13 \neq 13$

a)  $6 \times 1 = 6$

True or false?

**true**

b)  $1 \times 4 = 4$

True or false?

c)  $1 \times 1 = 2$

True or false?

d)  $9 \times 1 = 9$

True or false?

e)  $15 \div 1 = 1$

True or false?

f)  $3 \div 1 = 3$

True or false?

g)  $\boxed{\phantom{00}} \times 1 = 2$

h)  $\boxed{\phantom{00}} \times 1 = 7$

i)  $4 \times 1 = \boxed{\phantom{00}}$

j)  $5 \times \boxed{\phantom{00}} = 5$

k)  $4 \div \boxed{\phantom{00}} = 4$

l)  $8 \div \boxed{\phantom{00}} = 1$

m) Which expression equals 4?

- A)  $1 \times 4$
- B)  $1 + 4$
- C)  $1 \div 4$

n) Which expression equals 5?

- A)  $1 + 5$
- B)  $1 \div 5$
- C)  $1 \times 5$

o) Which expression equals 12?

- A)  $1 + 12$
- B)  $1 \div 12$
- C)  $1 \times 12$

p) Which expression equals 6?

- A)  $1 \div 6$
- B)  $1 - 6$
- C)  $1 \times 6$

q) Which expression equals 10?

- A)  $1 \div 10$
- B)  $10 \times 1$
- C)  $10 + 1$

r) Which expression equals 17?

- A)  $17 \times 17$
- B)  $17 \div 1$
- C)  $1 + 17$

## Skill 13.5 Using 'order of operations' involving + and/or - and × and/or ÷

MM3.2 1 1 2 2 3 3 4 4  
MM4.1 1 1 2 2 3 3 4 4

### Only + and/or -

- Add (+) and/or subtract (−) from left to right.

### Only × and/or ÷

- Multiply (×) and/or divide (÷) from left to right.

**Q.**  $8 - 2 - 5 + 6 =$

**A.** 
$$\begin{aligned}8 - 2 - 5 + 6 &= \\&= 6 - 5 + 6 \\&= 1 + 6 \\&= 7\end{aligned}$$

Start with 8 and subtract 2.  
The result is 6.  
Then subtract 5 from 6.  
The result is 1.  
Finally add 6 to the 1.

**a)**  $8 + 2 + 4 =$

$$= 10 + 4 = \boxed{14}$$

**b)**  $6 + 5 - 3 =$

$$= \boxed{\phantom{00}}$$

**c)**  $14 - 7 - 6 =$

$$= \boxed{\phantom{00}}$$

**d)**  $7 - 5 + 9 =$

$$= \boxed{\phantom{00}}$$

**e)**  $19 - 8 + 1 =$

$$= \boxed{\phantom{00}}$$

**f)**  $16 - 2 + 5 =$

$$= \boxed{\phantom{00}}$$

**g)**  $4 + 6 + 3 =$

$$= \boxed{\phantom{00}}$$

**h)**  $13 - 7 - 4 =$

$$= \boxed{\phantom{00}}$$

**i)**  $5 + 8 - 9 =$

$$= \boxed{\phantom{00}}$$

**j)**  $6 + 5 + 1 - 2 =$

$$\begin{aligned}= 11 + 1 - 2 \\= 12 - 2 = \boxed{10}\end{aligned}$$

**k)**  $8 - 4 + 3 + 2 =$

$$= \boxed{\phantom{00}}$$

**l)**  $9 + 7 - 5 - 1 =$

$$= \boxed{\phantom{00}}$$

**m)**  $7 + 3 + 5 - 6 =$

$$= \boxed{\phantom{00}}$$

**n)**  $5 - 2 + 7 - 5 =$

$$= \boxed{\phantom{00}}$$

**o)**  $9 - 3 - 2 - 1 =$

$$= \boxed{\phantom{00}}$$

**p)**  $2 \times 5 \times 3 =$

$$= 10 \times 3 = \boxed{30}$$

**q)**  $5 \times 3 \div 3 =$

$$= \boxed{\phantom{00}}$$

**r)**  $16 \div 4 \div 2 =$

$$= \boxed{\phantom{00}}$$

**s)**  $5 \times 4 \div 4 =$

$$= \boxed{\phantom{00}}$$

**t)**  $18 \div 6 \div 3 =$

$$= \boxed{\phantom{00}}$$

**u)**  $7 \times 2 \div 7 =$

$$= \boxed{\phantom{00}}$$

**v)**  $4 \times 2 \times 2 =$

$$= \boxed{\phantom{00}}$$

**w)**  $2 \times 9 \div 6 =$

$$= \boxed{\phantom{00}}$$

**x)**  $20 \div 5 \div 2 =$

$$= \boxed{\phantom{00}}$$

**INVERSE OPERATIONS + and -**

$+ 5 - 5 = 0$

*Subtraction of a number undoes addition of that same number.*

Example:  $17 + 5 - 5 = 17 + 0 = 17$   
because adding 5 and then subtracting 5 gives 0.

**INVERSE OPERATIONS  $\times$  and  $\div$** 

$\times 8 \div 8 = 1$

*Division by a number undoes multiplication by that same number.*

Example:  $6 \times 8 \div 8 = 6 \times 1 = 6$   
because multiplying by 8 and then dividing by 8 gives 1.

- Keep the number unchanged when it is followed by two inverse operations applied to the same number.

Q.  $24 + 9 - 9 =$

A.  $24 + 9 - 9 =$   
 $= 24 + 0$   
 $= 24$

Subtracting 9 undoes adding 9  
OR  
+ 9 and - 9 cancel each other.  
24 remains unchanged.

a)  $43 + 12 - 12 =$

*cancel each other*  
 $= 43 + 0$     = 43

b)  $31 - 6 + 6 =$

$=$  ..... =

c)  $17 + 3 - 3 =$

$=$  ..... =

d)  $15 + 8 - 8 =$

$=$  ..... =

e)  $23 - 19 + 19 =$

$=$  ..... =

f)  $24 - 7 + 7 =$

$=$  ..... =

g)  $20 + 13 - 13 =$

$=$  ..... =

h)  $18 - 9 + 9 =$

$=$  ..... =

i)  $21 - 10 + 10 =$

$=$  ..... =

j)  $20 \div 4 \times 4 =$

*cancel each other*  
 $= 20 \div 1$     =

k)  $14 \times 2 \div 2 =$

$=$  ..... =

l)  $25 \div 5 \times 5 =$

$=$  ..... =

m)  $16 \times 4 \div 4 =$

$=$  ..... =

n)  $45 \times 2 \div 2 =$

$=$  ..... =

o)  $32 \div 8 \times 8 =$

$=$  ..... =

p)  $9 \times 7 \div 7 =$

$=$  ..... =

q)  $18 \times 3 \div 3 =$

$=$  ..... =

r)  $24 \div 6 \times 6 =$

$=$  ..... =

## Skill 13.7 Using 'order of operations' involving single $\times$ or $\div$ and $+$ or $-$

MM3.2 1 1 2 2 3 3 4 4  
MM4.1 1 1 2 2 3 3 4 4

- Use the order of operations rules: First multiply ( $\times$ ) or divide ( $\div$ ).  
Finally add ( $+$ ) or subtract ( $-$ ).

**Q.**  $6 + 12 \div 3 =$

**A.**  $6 + 12 \div 3 =$  *first divide*  
 $= 6 + 4$  *then work from left to right*  
 $= 10$

First do 12 divided by 3.  
The result is 4.  
Then add 6 and 4.

**a)**  $21 \div 3 - 2 =$

$$= 7 - 2 = \boxed{5}$$

**b)**  $4 + 3 \times 3 =$

$$= \boxed{\phantom{00}}$$

**c)**  $6 \times 2 + 8 =$

$$= \boxed{\phantom{00}}$$

**d)**  $15 \div 5 - 2 =$

$$= \boxed{\phantom{00}}$$

**e)**  $2 \times 5 - 4 =$

$$= \boxed{\phantom{00}}$$

**f)**  $6 + 3 \times 5 =$

$$= \boxed{\phantom{00}}$$

**g)**  $6 + 9 \div 3 =$

$$= \boxed{\phantom{00}}$$

**h)**  $18 \div 2 + 4 =$

$$= \boxed{\phantom{00}}$$

**i)**  $3 \times 4 + 7 =$

$$= \boxed{\phantom{00}}$$

**j)**  $13 - 3 \times 3 =$

$$= \boxed{\phantom{00}}$$

**k)**  $4 \times 4 - 7 =$

$$= \boxed{\phantom{00}}$$

**l)**  $15 - 10 \div 5 =$

$$= \boxed{\phantom{00}}$$

**m)**  $21 \div 7 - 1 =$

$$= \boxed{\phantom{00}}$$

**n)**  $8 + 12 \div 4 =$

$$= \boxed{\phantom{00}}$$

**o)**  $15 - 5 \times 2 =$

$$= \boxed{\phantom{00}}$$

**p)**  $18 - 12 \div 2 =$

$$= \boxed{\phantom{00}}$$

**q)**  $16 \div 4 + 4 =$

$$= \boxed{\phantom{00}}$$

**r)**  $18 \div 6 - 3 =$

$$= \boxed{\phantom{00}}$$

**s)**  $8 + 28 \div 4 =$

$$= \boxed{\phantom{00}}$$

**t)**  $9 \times 6 - 3 =$

$$= \boxed{\phantom{00}}$$

**u)**  $24 - 12 \div 4 =$

$$= \boxed{\phantom{00}}$$

## Skill 13.8 Using 'order of operations' involving brackets ()

MM3.2 1 1 2 2 3 3 4 4  
MM4.1 1 1 2 2 3 3 4 4

- Use the order of operations rules: First evaluate inside the brackets.  
Then multiply ( $\times$ ) and/or divide ( $\div$ ) from left to right.  
Finally add (+) and/or subtract (-) from left to right.

**Q.**  $9 + 12 \div (9 - 5) =$

**A.**  $9 + 12 \div (9 - 5) =$   
 $= 9 + 12 \div 4$  *first brackets*  
 $= 9 + 3$  *then divide*  
 $= 12$  *then work from left to right*

Simplify inside the brackets and subtract 5 from 9.  
The result is 4.  
Then divide 12 by 4.  
The result is 3.  
Finally add 9 and 3.

**a)**  $7 \times (4 - 2) =$

$$= 7 \times 2 = \boxed{14}$$

**b)**  $9 - (4 + 3) =$

$$= \quad = \boxed{\phantom{00}}$$

**c)**  $7 - (5 - 2) =$

$$= \quad = \boxed{\phantom{00}}$$

**d)**  $10 - (9 - 2) =$

$$= \quad = \boxed{\phantom{00}}$$

**e)**  $(4 + 4) \times 3 =$

$$= \quad = \boxed{\phantom{00}}$$

**f)**  $15 \div (5 - 2) =$

$$= \quad = \boxed{\phantom{00}}$$

**g)**  $7 \times (6 - 2) =$

$$= \quad = \boxed{\phantom{00}}$$

**h)**  $(18 + 6) \div 3 =$

$$= \quad = \boxed{\phantom{00}}$$

**i)**  $28 \div (1 + 6) =$

$$= \quad = \boxed{\phantom{00}}$$

**j)**  $5 \times (11 - 8) =$

$$= \quad = \boxed{\phantom{00}}$$

**k)**  $(13 - 7) \div 2 =$

$$= \quad = \boxed{\phantom{00}}$$

**l)**  $42 \div (5 + 2) =$

$$= \quad = \boxed{\phantom{00}}$$

**m)**  $(12 - 7) \times 4 =$

$$= \quad = \boxed{\phantom{00}}$$

**n)**  $(5 + 8) \times 2 =$

$$= \quad = \boxed{\phantom{00}}$$

**o)**  $(12 - 9) \times 5 =$

$$= \quad = \boxed{\phantom{00}}$$

**p)**  $8 + (5 + 1) \div 2 =$

$$= 8 + 6 \div 2$$

$$= 8 + 3 = \boxed{11}$$

**q)**  $14 - 6 - (5 + 3) =$

$$= \quad = \boxed{\phantom{00}}$$

**r)**  $15 \div 3 - (2 + 2) =$

$$= \quad = \boxed{\phantom{00}}$$

**s)**  $9 + (7 - 4) \times 3 =$

$$= \quad = \boxed{\phantom{00}}$$

**t)**  $18 \div (9 - 3) + 2 =$

$$= \quad = \boxed{\phantom{00}}$$

**u)**  $9 + 3 \times (8 - 4) =$

$$= \quad = \boxed{\phantom{00}}$$