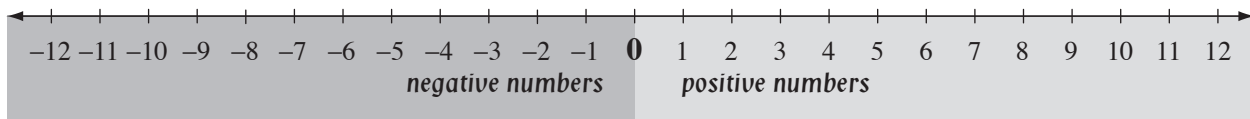# 13. [Integers]

MM4.2 **11** 22 33 44
MM5.1 **11** 22 33 44

- Use a number line.
  Hint: Numbers decrease as you move to the left or down and increase as you move to the right or up.

NUMBER LINE
A negative number is always smaller than a positive number.

–12 –11 –10 –9 –8 –7 –6 –5 –4 –3 –2 –1 **0** 1 2 3 4 5 6 7 8 9 10 11 12
negative numbers   *positive numbers*

15°C
10°C
5°C
0°C
-5°C
-10°C
-15°C

ALTITUDE

An altitude is lower when further down, below sea level (BSL) and higher when further up, above sea level (ASL).

Above Sea Level (ASL)
800 m
600 m
400 m
200 m
0
–200 m
–400 m
–600 m
–800 m
Sea Level
Below Sea Level (BSL)
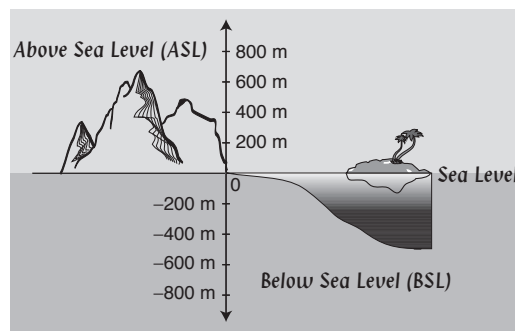
TEMPERATURE

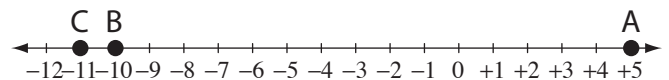Temperatures below zero are lower than temperatures above zero.
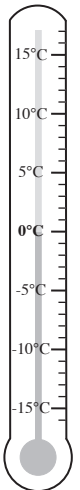
Q. Who won the 2010 Women's British Open Golf Tournament?
[Hint: In golf the lowest score wins.]

A) +5 K. Webb
B) −10 K. Hull
C) −11 Y. Tseng

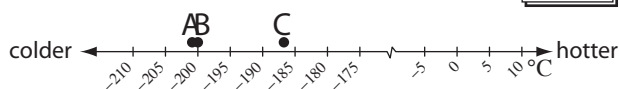**A.** *C*   Find the lowest score to determine the winner.

C B   A
–12 –11 –10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 +1 +2 +3 +4 +5

---

a) Which of Saturn's moons has the highest temperature?

A) −201ºC Enceladus
B) −200ºC Mimas
C) −187ºC Tethys

C

colder ← –210 –205 –200 –195 –190 –185 –180 –175 –5 0 5 10 °C → hotter
A B   C

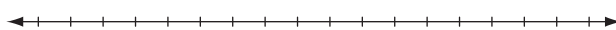**b)** Which temperature for oxygen is higher?

A) −183ºC boiling point
B) −218ºC melting point

°C
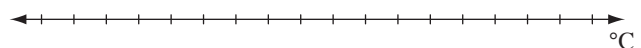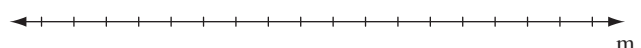
c) Who won the 2010 British Open Golf Tournament?
[Hint: In golf the lowest score wins.]

A) −16 L. Oosthuizen
B) +3 P. Senior
C) −2 R. Allenby

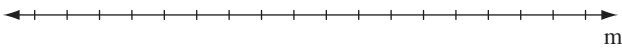**d)** Which body of water is at the lowest altitude?

A) −28 m   Caspian Sea
B) −408 m Dead Sea
C) −15 m   Lake Eyre

m

e) Which location has the lowest altitude?

A) 3 m above sea level
Amsterdam (Netherlands)
B) 133 m below sea level
Qattara Depression (Egypt)
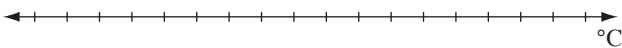C) 2430 m above sea level
Machu Picchu (Peru)

m

f) Which location has the highest altitude?

A) 10 m below sea level
Laguna Salada (Mexico)
B) 7 m below sea level
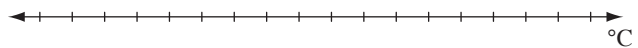Lammefjord (Denmark)
C) 19 m above sea level
Vatican City (Italy)

m
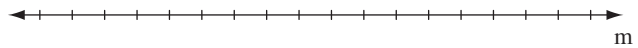
g) Which location recorded the lowest temperature?

A) −25.6°C   Kabul
B) +14.1°C   Christmas Island
C) −15.2°C   La Paz

°C

h) Which continent has the lowest recorded temperature?

A) −63°C   North America
B) −23°C   Australia
C) −55°C   Europe

°C

i) Arrange in ascending order:
$-4, -7, 5, -3, 3$

−7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7

−7, −4, −3, 3, 5
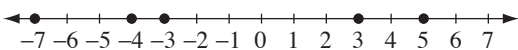
j) Arrange in order from largest to smallest:
$0, 8, -9, 6, -4$

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7
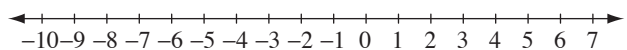
k) Arrange in descending order:
$-10, 8, 1, -8, 4$

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7
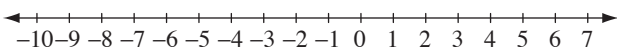
l) Arrange in order from smallest to largest:
$-2, -6, 0, -3, 5$

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7
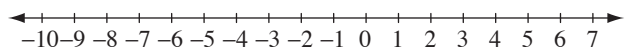
m) Arrange in order from coldest to warmest:
$2°C, -3°C, 4°C, -5°C$

−7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 °C
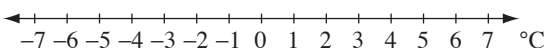
n) Arrange in order from warmest to coldest:
$-1°C, -5°C, 5°C, -3°C$

−7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 °C
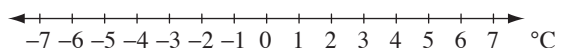
- Use a number line.
  Hint: A negative number is always smaller than a positive number.
  The larger the negative number the lesser the value, e.g. –9 is less than (<) –2
  The smaller the negative number the greater the value, e.g. –4 is greater than (>) –6

Q. Use < or > to make a true statement.

$$3 \boxed{\phantom{<}} -8$$

A. *3 > –8*   3 is greater than –8

–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7

less than <   > greater than

a) Use < or > to make a true statement.

a negative number is less than a positive number

$$-3 \boxed{<} 4$$

–7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7

b) Use < or > to make a true statement.

$$-5 \boxed{\phantom{<}} 0$$

–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7

c) Use < or > to make a true statement.

$$-4 \boxed{\phantom{<}} -9$$

–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7
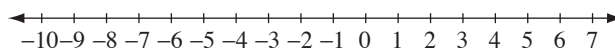
d) Use < or > to make a true statement.

$$-6 \boxed{\phantom{<}} 3$$

–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7
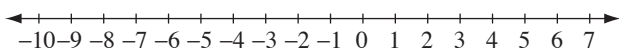
e) Use < or > to make a true statement.

$$2 \boxed{\phantom{<}} -1$$

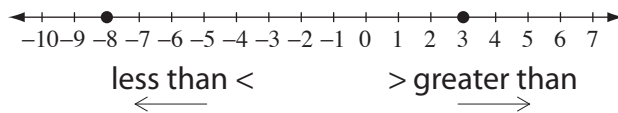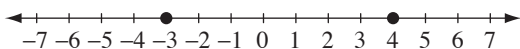–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7
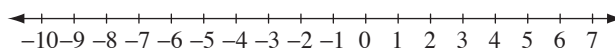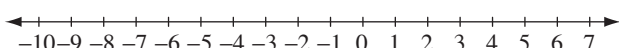
f) Use < or > to make a true statement.

$$-3 \boxed{\phantom{<}} -7$$

–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7

g) Use < or > to make a true statement.

$$-9 \boxed{\phantom{<}} 0$$

–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7
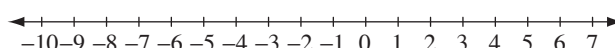
h) Use < or > to make a true statement.

$$3 \boxed{\phantom{<}} -5$$

–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7
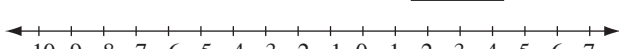
i) Use < or > to make a true statement.

$$4 \boxed{\phantom{<}} -7$$
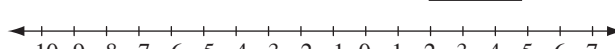
–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7

j) Use < or > to make a true statement.

$$-4 \boxed{\phantom{<}} -2$$

–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7
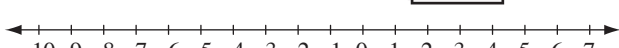
k) Use < or > to make a true statement.

$$-8 \boxed{\phantom{<}} -5$$

–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7

l) Use < or > to make a true statement.

$$-2 \boxed{\phantom{<}} -4$$

–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7

www.mathsmate.co.nz   © Maths Mate 4.2/5.1 Skill Builder 13
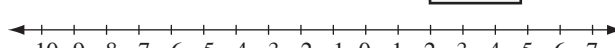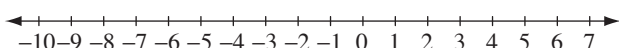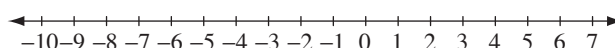
Skill 13.3   Modelling integer subtraction on a number line.

MM4.2 11 22 33 44
MM5.1 11 22 33 44

- Determine the value of each mark on the number line.
- Count the number of spaces between the integers using the number line.
  Hint: Use short cuts such as: counting to zero, counting by tens.

Q. How many units between 6 and −4?

$$-7\ -6\ -5\ -4\ -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7$$

A. *10*

10 9 8 7 6 5 4 3 2 1

$$-7\ -6\ -5\ -4\ -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7$$
Start
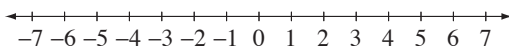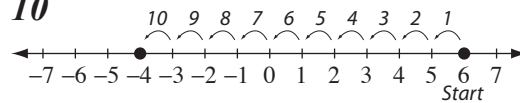
a) How much cooler is it in Geneva than Mombasa?

6   26

Geneva   Mombasa

Count to zero first
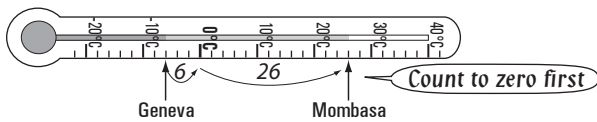
6 + 26 = **32** °C

b) If Karrie Webb scores a triple bogie and Greg Norman scores an eagle, what is the difference between their scores?

albatross eagle birdie **par** bogie double bogie triple bogie quadruple bogie

$$-3\ -2\ -1\ 0\ +1\ +2\ +3\ +4$$

c) How many units between 5 and −4?

$$-7\ -6\ -5\ -4\ -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7$$
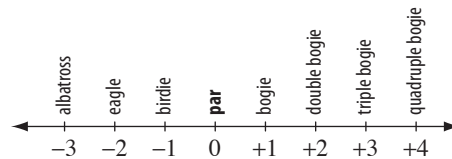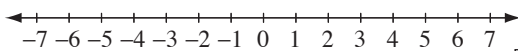
d) How many units between −5 and 3?

$$-7\ -6\ -5\ -4\ -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7$$
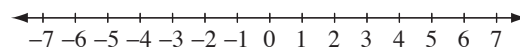
e) How many units between −9 and 2?

$$-10\ -9\ -8\ -7\ -6\ -5\ -4\ -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7$$
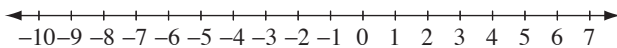
f) How many units between 6 and −7?

$$-10\ -9\ -8\ -7\ -6\ -5\ -4\ -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7$$
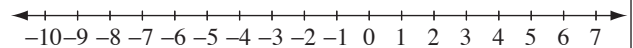
g) What is the difference between the highest and the lowest temperatures recorded in Dunedin, New Zealand?

Low   High
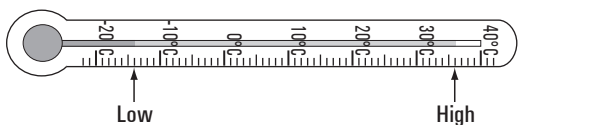
= ☐ °C

h) What is the difference between the highest and the lowest temperatures recorded in Rome, Italy?

low   high
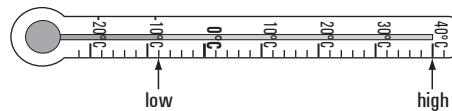
= ☐ °C

i) What is the time difference in hours between Denver and Cape Town?

LA Denver Chicago New York Greenland Sao Paulo Azores London Paris Cape Town Moscow Tehran Karachi Calcutta Jakarta

$$-7\ -6\ -5\ -4\ -3\ -2\ -1\ 0\ +1\ +2\ +3\ +4\ +5\ +6\ +7\ h$$
GMT
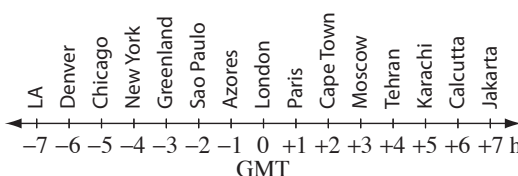
j) What is the time difference in hours between Karachi and New York?
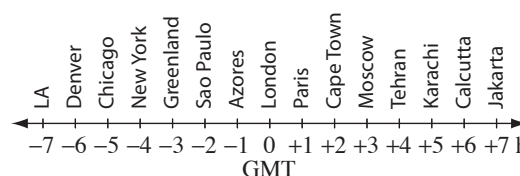
LA Denver Chicago New York Greenland Sao Paulo Azores London Paris Cape Town Moscow Tehran Karachi Calcutta Jakarta

$$-7\ -6\ -5\ -4\ -3\ -2\ -1\ 0\ +1\ +2\ +3\ +4\ +5\ +6\ +7\ h$$
GMT

MM4.2 11 22 **3** 44
MM5.1 1**1** 22 33 44

- Visualise the position of the values on a number line.
- Translate the words to number sentences.
- Add the numbers ignoring their signs.
  Hint: Taking away negative 5 is the same as adding positive 5.
  $$0 - (-5) = +5$$

Q. In Vienna (Austria) the highest recorded temperature is 39°C and the lowest is −25°C. What is the temperature difference?

A. $39 - (-25)$
$= 39 + 25$
$= \mathbf{64°C}$

Instead of subtracting negative 25, add positive 25 to 39.

temperature difference
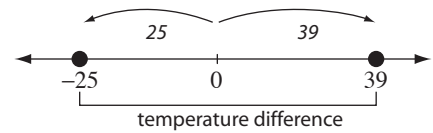
a) The Bay of Fundy, Canada has a high tide of 8.5 m and a low tide of −8.5 m. What is the tidal range for the Bay of Fundy?

tidal range (m)
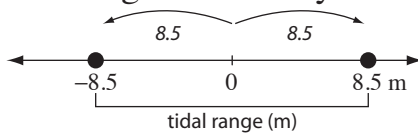
$= 8.5 - (-8.5)$

$8.5 + 8.5$ = **17 m**

b) The lowest point in Japan is Lake Hachirogata at −4 m and the highest point is Mt Fujiyama at 3776 m. What is the height difference?

=
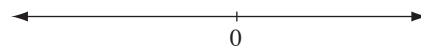
= ___ m

c) Sparrow Hills station is the highest station in the Russian metro rail system with an altitude of 220 m above sea level. Park Pobedy is the lowest station at 90 m below sea level. What is their height difference?

= ___ m

d) In Reykjavik (Iceland) the highest recorded temperature is 26°C and the lowest is −25°C. What is the temperature difference?

= ___ °C

e) In Luxembourg the highest recorded temperature is 38°C and the lowest is −23°C. What is the temperature difference?

= ___ °C

f) In Shanghai (China) the highest recorded temperature is 40°C and the lowest is −12°C. What is the temperature difference?

= ___ °C

g) The lowest point on the African continent is −156 m at Lake Assal and the highest is 5895 m at Mt Kilimanjaro. What is the height difference?

= ___ m

h) The highest point in Europe is 5642 m at Mt Elbrus and the lowest is −22 m in the Caspian Sea. What is the height difference in Europe?

= ___ m

- Start at the given point on the number line.
- Count up or down the number of spaces as directed.

Q. From homewares Marion rides the elevator down 6 levels and up 3 levels. At what level is Marion now?

5 ladies
4 homewares
3 books/toys
2 mens & childrens
1 cosmetics/shoes
0 food court
−1 carpark (A)
−2 carpark (B)
−3 carpark (C)
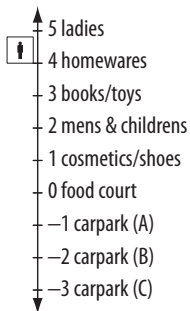
A. *down 6 levels (add −6)*
up 3 levels (add +3)
⇒ ***Cosmetics/shoes***

5
Start 4 homewares
3
2
down 6 1 cosmetics/shoes
0
up 3 −1
−2
−3
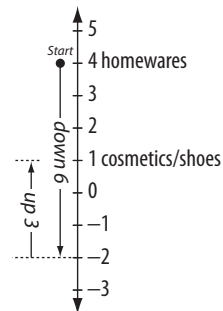
a) From level 1 Hutch rides the elevator up 4 levels and down 8. At what level is Hutch now?
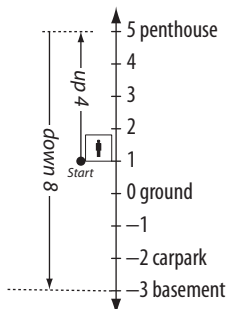
up 4 levels (add +4)

down 8 levels (add −8)   **basement**

5 penthouse
4
up 4 3
2
1
down 8 Start
0 ground
−1
−2 carpark
−3 basement

b) From the carpark Kwong rides the elevator down 1 level and up 3 levels. At what level is Kwong now?

....

⇒

5 penthouse
4
3
2
1
0 ground
−1
−2 carpark
−3 basement
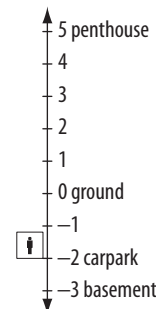
c) A nurse starts in cardiac ward, goes down 6 levels and then up 3 levels. Where does she finish?

....

⇒

5 cardiac
4 surgical
3 orthopaedics
2 oncology
1 gyneacology
0 emergency
−1 cafeteria
−2 carpark
−3 basement
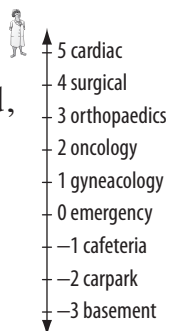
d) A termite entered his tower via the central vent and went to the main nest. How far did the termite travel?

....

⇒   **m**

All distances in metres
5 central vent
4
3
2 side vent
1
0 ground level
−1 main nest
−2
−3 food storage
−4 queen's cell
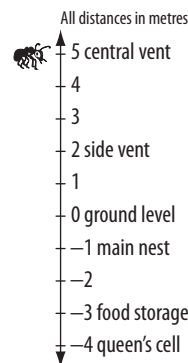
e) From carpark (A) Todd rides the elevator down 2 levels and up 7 levels. At what level is Todd now?

....

⇒

5 ladies
4 homewares
3 books/toys
2 mens & childrens
1 cosmetics/shoes
0 food court
−1 carpark (A)
−2 carpark (B)
−3 carpark (C)
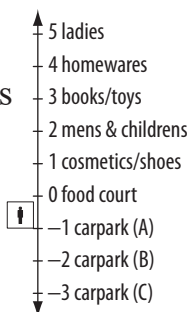
f) A snorkeller at the reef surfaces for lunch on the pier and then goes back to the reef. How far does he travel?

....

⇒   **m**

All distances in metres
3 pier
2
1 platform
0 sea level
−1
−2
−3
−4
−5 reef

Skill 13.6   Solving word problems involving two or more integers.

MM4.2 11 22 33 **4** 4
MM5.1 11 **22** 33 44
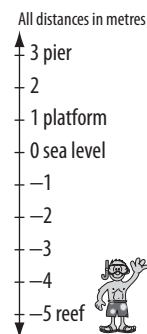
- Start at the given point.
- Work in the given order.
- Visualise the position of the values on a number line.

   Hint: Positive words: up,   above,   over,   forward, advance, gained,   earned,   later
   Negative words: down,   below,   under,   backward, retreat,   lost,   owed,   earlier

---

Q. During a football game the ball advanced 11 m, retreated 7 m and then advanced 2 m. Where did the ball finish in relation to its starting point?

A. *Start: 0 + 11 − 7 + 2*
    = 4 + 2
    = +6 m

---
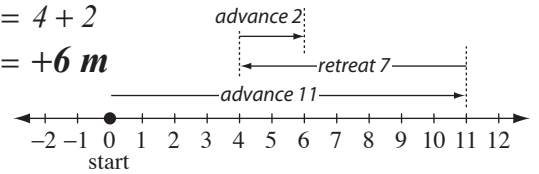
a) If Pip had \$400 and spent \$950, what is her bank balance?

$= 400 - 950$ $= \boxed{-\$550}$

spent \$950
−\$550   0   \$400
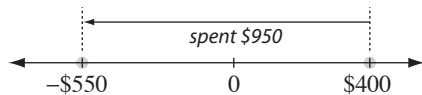
b) Harry owed \$350. If he earned \$420, how much does Harry now have?

$=$ $= \boxed{}$

c) Chan owes \$420. If he earned \$280, what is Chan's bank balance?

$=$ $= \boxed{}$

d) Carbon dioxide boils at −78°C. At 21°C below this, carbon dioxide solidifies. At what temperature does carbon dioxide solidify?

$=$ $= \boxed{\ °C}$

e) The Persians destroyed the original Acropolis in 480 BC. Pericles rebuilt it 31 years later. What year was that?

$=$ $= \boxed{}$

f) Tutenkhamun reigned for 9 years up until 1323 BC. What year did Tutenkhamun come to the throne?

$=$ $= \boxed{}$

g) Oxygen boils at −183°C. At 35°C below this, oxygen solidifies. What is the temperature of solid oxygen?

$=$ $= \boxed{\ °C}$

h) Helium boils at −269°C. At 3°C below this, helium solidifies. At what temperature does helium solidify?

$=$ $= \boxed{\ °C}$

i) You bought \$1000 worth of stock. After the first year you lost \$480, but after the second year you gained \$220. What is the current value of your stock?

$=$ $= \boxed{}$

j) A bear weighs 67 kg. During hibernation it loses 20 kg. After hibernation it gains 14 kg. What does the bear weigh now?

$=$ $= \boxed{\ kg}$

Hint: Every number has a sign attached to it, so if there is no sign, the number is positive. These signs should not be confused with the operations of addition and subtraction.

Using a number line
- Start at 0.
- When the number is "+" move that many to the right.
- When the number is "−" move that many to the left.

Using Addition Rules

Addition Rules
same signs:
Add the numbers, ignoring their signs.
Keep that sign.

Addition Rules
different signs:
Subtract the numbers, ignoring their signs.
Keep the sign of the larger number.

Example:
$(+4) + (+3) = +(4 + 3) = +7 = 7$ *or simply*
$4 + 3 = 7$
$(-5) + (-8) = -(5 + 8) = -13$ *or simply*
$-5 + -8 = -13$

Example:
$(-9) + (+3) = -(9 - 3) = -6$ *or simply*
$-9 + 3 = -6$
$(-4) + (+11) = +(11 - 4) = +7 = 7$ *or simply*
$-4 + 11 = 7$

Q. $-5 + 7 =$

A. $-5 + 7 =$
$= +(7 - 5)$
$= 2$

Starting at 0 go 5 units to the left. From this point, move 7 units right. You stop at positive 2.

Negative Integers | Zero | Positive Integers

+1 +2 +3 +4 +5 +6 +7 +8 +9 +10 +11 +12 +13 +14

−14 −13 −12 −11 −10 −9 −8 −7 −6 −5 −4 −3 −2 −1 **0** 1 2 3 4 5 6 7 8 9 10 11 12 13 14

-5
$+7$

---
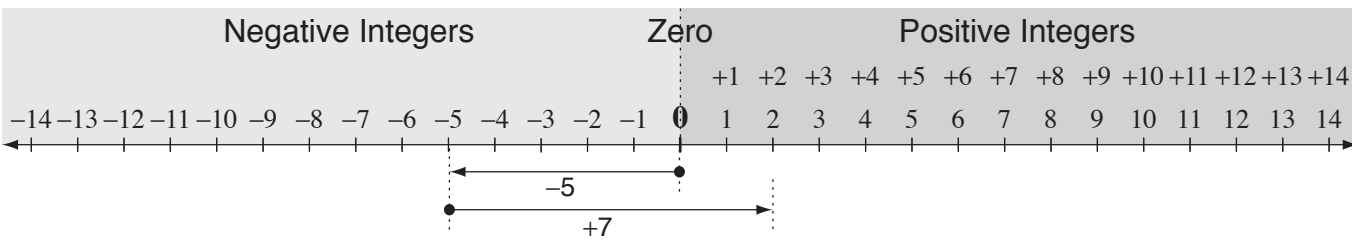
a) $-2 + (-3) =$

$= -(2 + 3)$ $= \boxed{-5}$

both negative keep "−"   same signs, add

b) $-4 + 3 =$

$=$

c) $-8 + 6 =$

$=$

d) $8 + (-5) =$

$=$

e) $2 + (-6) =$

$=$

f) $5 + (-3) =$

$=$

g) $-2 + 4 =$

$=$

h) $9 + (-2) =$

$=$

i) $-4 + (-2) =$

$=$

j) $-8 + 3 =$

$=$

k) $-2 + (-6) =$

$=$

l) $-3 + (-6) =$

$=$

> *Hint:* *Every number has a sign attached to it, so if there is no sign, the number is positive.*
> *These signs should not be confused with the operations of addition and subtraction.*

Using a number line
- Start at 0.
- When the number is "+" move that many to the right.
- When the number is "−" move that many to the left.

Using Addition Rules
- Consider subtracting an integer as adding its opposite.
  So change the number to be subtracted to its opposite. Example: $8 - (-2) = 8 + (+2)$
- Then apply the addition rules.

Addition Rules	**Addition Rules**
same signs:	different signs:
Add the numbers, ignoring their signs.	Subtract the numbers, ignoring their signs.
Keep that sign.	Keep the sign of the larger number.

Example:

$(-9) - (-3) = (-9) + (+3) = -(9-3) = -6$

or simply $-9 + 3 = -6$

$(-4) - (-11) = (-4) + (+11) = +(11-4)$
$= +7 = 7$ *or simply* $-4 + 11 = 7$

Example:

$(-5) - (+8) = (-5) + (-8) = -(5+8) = -13$

or simply $-5 + -8 = -13$

$(+4) - (-3) = (+4) + (+3) = +(4+3) = +7 = 7$

or simply $4 + 3 = 7$

Q. $-3 - 6 =$

start at −3, move backward 6

A. $-3 - 6 =$
$= -3 + (-6)$
$= -(3 + 6)$
$= -9$

Negative 3 take away positive 6 is the same as negative 3 plus negative 6.

OR Using a number line:

Starting at 0 go 3 units to the left.
From this point, move 6 units left. You stop at negative 9.

Negative Integers	Zero	Positive Integers

+1 +2 +3 +4 +5 +6 +7 +8 +9 +10 +11 +12 +13 +14

−14 −13 −12 −11 −10 −9 −8 −7 −6 −5 −4 −3 −2 −1 **0** 1 2 3 4 5 6 7 8 9 10 11 12 13 14

−6   −3

---
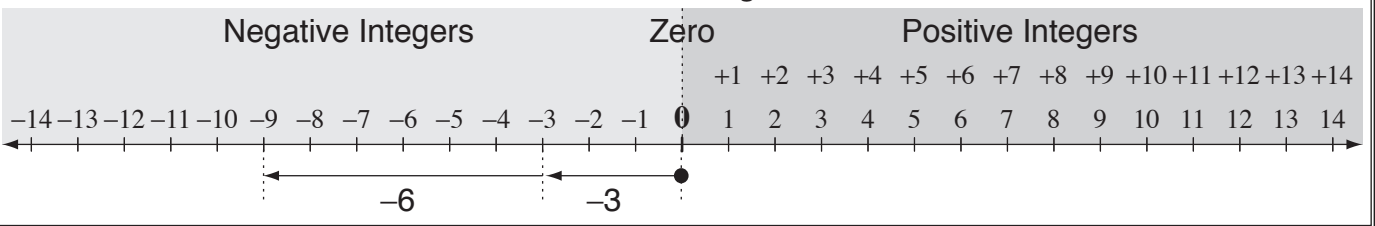
a) $1 - 7 =$ *subtract 7 means add −7*

$= +1 + (-7)$ *different signs, subtract*

$= -(7 - 1)$ $= \boxed{-6}$

b) $0 - 8 =$

$=$

$= \boxed{}$

c) $4 - 8 =$

$=$

$= \boxed{}$

d) $-3 - 5 =$

$=$

$= \boxed{}$

e) $-9 - 2 =$

$=$

$= \boxed{}$

f) $2 - (-1) =$

$=$

$= \boxed{}$

g) $3 - (-4) =$

  = ...

  = ...   = ☐

h) $-8 - (-4) =$

  = ...

  = ...   = ☐

i) $-2 - (-2) =$

  = ...

  = ...   = ☐

j) $-8 - 5 =$

  = ...

  = ...   = ☐

k) $9 - (-6) =$

  = ...

  = ...   = ☐

l) $-7 - (-3) =$

  = ...

  = ...   = ☐

m) $2 - 11 =$

  = ...

  = ...   = ☐

n) $5 - (+7) =$

  = ...

  = ...   = ☐

o) $-8 - (-2) =$

  = ...

  = ...   = ☐

p) $0 - (-5) =$

  = ...

  = ...   = ☐

q) $-6 - (+2) =$

  = ...

  = ...   = ☐

r) $-3 - 7 =$

  = ...

  = ...   = ☐

s) $5 - (-2) =$

  = ...

  = ...   = ☐

t) $-4 - (-10) =$

  = ...

  = ...   = ☐

u) $8 - (-9) =$

  = ...

  = ...   = ☐

v) $-6 - 10 =$

  = ...

  = ...   = ☐

w) $3 - 9 =$

  = ...

  = ...   = ☐

x) $-1 - 8 =$

  = ...

  = ...   = ☐

y) $0 - 12 =$

  = ...

  = ...   = ☐

z) $10 - (+3) =$

  = ...

  = ...   = ☐

A) $-7 - 2 =$

  = ...

  = ...   = ☐

Skill 13.9   Multiplying integers.

- Multiply the integers ignoring the signs.
- Determine the sign of the result using the multiplication rules.

Multiplcation Rules	**Multiplcation Rules**
same signs: positive $\times$ positive = positive	different signs: positive $\times$ negative = negative
negative $\times$ negative = positive	negative $\times$ positive = negative

Example: $-9 \times (-3)$
$= 27$ $-- = +$

Example: $9 \times (-3)$
$+9 \times (-3)$
$= -27$ $+ - = -$

Q. $-4 \times (-7) =$ **A.** $-4 \times (-7) =$ $4 \times 7 = 28$
                        $= 28$ $-- = +$ Same signs, both negative
                                $\Rightarrow$ positive result.
                                $\Rightarrow +28$

---

a) $-6 \times 7 =$ $-+=-$ $\boxed{-42}$

b) $-2 \times 6 =$

c) $-8 \times 3 =$

d) $3 \times (-5) =$

e) $2 \times (-9) =$

f) $-8 \times (-8) =$

g) $-8 \times (-5) =$

h) $-9 \times 4 =$

i) $5 \times (-6) =$

j) $7 \times (-8) =$

k) $-4 \times 6 =$

l) $-7 \times 7 =$

m) $3 \times (-9) =$

n) $-7 \times (-4) =$

o) $-6 \times (-3) =$

p) $-4 \times (-4) =$

q) $5 \times (-9) =$

r) $-8 \times (-2) =$

s) $-5 \times (-5) =$

t) $-4 \times 5 =$

u) $-9 \times 9 =$

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Skill 13.10 Dividing integers.

- Divide the integers ignoring the signs.
- Determine the sign of the result using the division rules.

Division Rules	**Division Rules**
same signs: positive ÷ positive = positive	different signs: positive ÷ negative = negative
negative ÷ negative = positive	negative ÷ positive = negative

Example: $-9 ÷ (-3)$
$= 3$ $- - = +$

Example: $9 ÷ (-3)$
$+9 ÷ (-3)$
$= -3$ $+ - = -$

Q. $-30 ÷ 6 =$

A. $-30 ÷ +6 =$
$= -5$ $- + = -$

$30 ÷ 6 = 5$
Different signs
⇒ negative result.
⇒ -5

---

a) $12 ÷ (-4) =$ $+ - = -$ -3

b) $27 ÷ (-3) =$

c) $-54 ÷ (-9) =$

d) $-72 ÷ (-12) =$

e) $-45 ÷ 9 =$

f) $-32 ÷ 8 =$

g) $-18 ÷ 2 =$

h) $-24 ÷ (-8) =$

i) $-63 ÷ 9 =$

j) $25 ÷ (-5) =$

k) $-56 ÷ (-7) =$

l) $-21 ÷ 7 =$

m) $-45 ÷ 5 =$

n) $-28 ÷ (-7) =$

o) $-54 ÷ 6 =$

p) $28 ÷ (-4) =$

q) $-35 ÷ (-7) =$

r) $-40 ÷ (-5) =$

s) $-36 ÷ 6 =$

t) $63 ÷ (-7) =$

u) $-36 ÷ 9 =$