

10. [Rates / Ratios]

Skill 10.1 Finding the unit rate and the unit price.

MM5.2 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

To find the unit rate

- Divide the two quantities to show how many of the first quantity correspond to one of the second quantity.

To find the unit price

- Divide the total amount by the number of items.

Q. 200 litres flow in 5 minutes =

litres per minute

A. *200 litres in 5 minutes* Simplify the fraction by 5.

$$= \frac{200 \text{ L}}{5 \text{ min}}$$

$$= \frac{200 \text{ L}}{5 \text{ min}}$$

$$= 40 \text{ litres per minute (L/min)}$$

a) 300 cars sold in 30 days =

cars per day

$$\frac{300 \text{ cars in 30 days}}{30 \text{ days}}$$

Simplify: $\div 30$

$$= 10 \text{ cars per day}$$

b) 420 m walked in 5 minutes =

m per minute

$$\frac{420 \text{ m in 5 minutes}}{5 \text{ minutes}}$$

$$=$$

$$=$$

c) 12°C drop in 6 hours =

°C per hour

$$\frac{12^\circ\text{C drop in 6 hours}}{6 \text{ hours}}$$

$$=$$

$$=$$

d) 600 beats in 10 minutes =

beats per minute

$$\frac{600 \text{ beats in 10 minutes}}{10 \text{ minutes}}$$

$$=$$

$$=$$

e) If 8 kg of apples are sold for \$10, what is the cost per kilogram?

$$\frac{\$10}{8 \text{ kg}}$$

$$=$$

$$= \text{ \$ } \text{ /kg}$$

f) The plumber charged \$231 for a 3-hour job. What was the charge per hour?

$$\frac{\$231}{3 \text{ hours}}$$

$$=$$

$$= \text{ /h}$$

g) Nina earns \$100 for an 8-hour shift. How much does she earn per hour?

$$\frac{\$100}{8 \text{ hours}}$$

$$=$$

$$= \text{ /h}$$

h) A 6-pack of exercise books cost \$9.60. What is the price per exercise book?

$$\frac{\$9.60}{6 \text{ books}}$$

$$=$$

$$= \text{ \$ }$$

Skill 10.2 Simplifying ratios.

MM5.2 **1** 1 22 33 44
MM6.1 **1** 1 22 33 44

- Write the quantities of the ratio with the same unit of measurement.

EITHER

- Find the largest number that divides evenly into each quantity of the ratio (Highest Common Factor).
- Divide each quantity by the HCF.

*Hints: The order of the quantities in a ratio matters.
'∴' means 'to'.*

Examples: The ratio of legs to ears in a dog is $4 : 2 = 2 : 1$
The ratio of ears to legs in a dog is $2 : 4 = 1 : 2$

OR

- Divide each quantity of the ratio by any factor until the ratio is reduced to its simplest form.

$$a : b = \frac{a}{b} \quad \text{Ratio}$$

Q. Simplify 80 min : 3 h **A.** $3 \text{ h} = 3 \times 60 \text{ min} = 180 \text{ min}$ **OR A.** $3 \text{ h} = 3 \times 60 \text{ min} = 180 \text{ min}$

$$\begin{aligned} &80 \text{ min} : 3 \text{ h} \\ &= \overset{4}{80} \text{ min} : \overset{9}{180} \text{ min} \\ &= 4 : 9 \end{aligned}$$

1 h = 60 min
HCF of 80 and 180 is 20 so $\div 20$
Ignore the units

$$\begin{aligned} &80 \text{ min} : 3 \text{ h} \\ &= 80 \text{ min} : 180 \text{ min} \\ &= \overset{4}{8} : \overset{9}{18} \\ &= 4 : 9 \end{aligned}$$

Simplify: $\div 10$
Simplify: $\div 2$

a) Simplify 600 mL : 0.2 L

1 L = 1000 mL
3 zeros, 3 places right

$$0.2 \text{ L} = 0.\overbrace{200}^{\text{3 zeros, 3 places right}} \times 1000 \text{ mL} = 200 \text{ mL}$$

$$\begin{aligned} &600 \text{ mL} : 200 \text{ mL} \\ &= 6 : 2 \end{aligned}$$

Simplify: $\div 100$
Simplify: $\div 2$

Answer: 3 : 1

b) Simplify 2 m : 50 cm

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.....

.....

.....

Answer:

c) Simplify 750 g : 1 kg

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.....

.....

.....

Answer:

d) Simplify 6 months : 4 years

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.....

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Answer:

e) Simplify 2 km : 3200 m

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.....

Answer:

f) Simplify \$24 : 300¢

.....

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.....

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Answer:

g) Simplify 0.5 kg : 2000 g : 4 kg

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.....

.....

Answer:

h) Simplify 50¢ : \$4.00 : \$2.50

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.....

.....

.....

Answer:

Skill 10.3 Finding the ratio of two or more quantities as a set : set comparison.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Write the ratio in words.
- Replace the words with numbers.
- Simplify the ratio:

EITHER

- Find the largest number that divides evenly into each quantity of the ratio (Highest Common Factor) and divide each quantity by the HCF.

Hint: The order of the quantities in a ratio matters.

OR

- Divide each quantity of the ratio by any factor until the ratio is reduced to simplest form.

Q. Ocean water represents 70% of the earth's surface and the rest is land. Find the ratio of water to land.

A. $land = 100\% - 70\% = 30\%$

water : land

$= 70\% : 30\%$

Simplify: $\div 10$

$= 7 : 3$

Ignore the % sign

a) An orchestra has 60 strings, 12 brass and 9 woodwinds instruments. What is the ratio of strings to brass to woodwinds instruments?

strings : brass: woodwind

$60 : 12 : 9$

Simplify: $\div 3$ $\overset{20}{60} : \overset{4}{12} : \overset{3}{9} = \boxed{20 : 4 : 3}$

b) Tin foil is made up of 88% tin, 4% copper and the rest lead. What is the ratio of tin to copper to lead in the foil?

:
:
:

$= \boxed{}$

c) Find the ratio of the average weight of a Blue Whale (120 t) to the average weight of a Humpback Whale (30 t).

:
:

$= \boxed{}$

d) Find the ratio of the Boeing 747 wingspan (64 m) to the Airbus A380 wingspan (80 m).

:
:

$= \boxed{}$

e) A ticket to the musical "The Boy from Oz" is \$90. A movie ticket to the Village cinemas is \$15. What is the ratio of musical to movie ticket prices?

:
:

$= \boxed{}$

f) To purchase a "Two Day Park Hopper" to Disneyland, Florida, costs \$96 for a child and \$116 for an adult. What is the ratio of an adult to child ticket prices?

:
:

$= \boxed{}$

g) Commercial butter is approximately 80% milk fat and 20% other components. What is the ratio of milk fat to other components?

:
:

$= \boxed{}$

h) Of the 76 seats in the Senate, 28 belong to the Labor Party. What is the ratio of Labor Party to other parties seats in the Senate?

:
:

$= \boxed{}$

Skill 10.4 Finding the ratio of two quantities as a subset : set comparison.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Write the ratio in words.
- Replace the words with numbers.
- Simplify the ratio:

EITHER

- Find the largest number that divides evenly into each quantity of the ratio (Highest Common Factor) and divide each quantity by the HCF.

Hint: The order of the quantities in a ratio matters.

OR

- Divide each quantity of the ratio by any factor until the ratio is reduced to simplest form.

- Q.** Of the 22 million Australian people, approximately 4 million live in Sydney. What is the ratio of Sydney's population to Australian population?

- A.** Sydney's population : Australian population

$$= 4 \text{ million} : 22 \text{ million} \quad \text{Ignore the units}$$

$$= \frac{4}{2} : \frac{22}{2}$$

$$= 2 : 11 \quad \text{Simplify: } \div 2$$

- a)** Of the 226 seats in the Australian Parliament, 76 are in the Senate. What is the ratio of Senate to Parliament seats?

Senate seats : Parliament seats

$$76 : 226$$

$$\text{Simplify: } \div 2 \quad \frac{76}{2} : \frac{226}{2} = 38 : 113$$

- b)** Land represents 30% of the earth's surface. Find the ratio of land to earth's surface.

:

:

=

- c)** What is the ratio of carbon to the total number of atoms in the ethane formula C_2H_6 ?

:

:

=

- d)** What is the ratio of hydrogen to the total number of atoms in the ethylene formula C_2H_4 ?

:

:

=

- e)** A viscose/polyester blouse has 44% polyester. What is the ratio of viscose to total composition?

:

:

=

- f)** Of the 2 L of cordial drink, 250 mL is concentrated cordial. Find the ratio of concentrated cordial to cordial drink.

:

:

=

- g)** Of the \$500 000 paid for the property, \$150 000 was for the block of land, and the rest was for building the house. Find the ratio of land to total property price.

:

:

=

- h)** Blood plasma makes up 55% of the human blood composition. Find the ratio of plasma to total blood components.

:

:

=

Skill 10.5 Finding the average speed.

MM5.2 1 1 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

$$\text{average speed } (v) = \frac{\text{distance travelled } (d)}{\text{time taken } (t)} \quad \text{OR} \quad v = \frac{d}{t}$$

- Write the formula for the average speed.
- Convert the given units into the required units if necessary. (see Maths Facts, page 455)
*Hints: If the speed must be calculated in km/h, convert the units for distance to km and convert the units for time to h.
Changing from smaller units into larger units, always divide by the conversion factor.
Changing from larger units into smaller units, always multiply by the conversion factor.*
- Substitute the values for distance and time into the formula.
- Evaluate and simplify.

Q. The marine green turtle was recorded swimming 480 km in 10 days. What was its average speed in km/h?

A. $v = \frac{d}{t}$
 where $t = 10 \text{ days} = 10 \times 24 \text{ h} = 240 \text{ h}$
 $v = \frac{480 \text{ km}}{240 \text{ h}}$ *Substitute into the formula*
 $= \frac{2480}{1240} \text{ km/h}$ *Simplify: $\div 240$*
 $= 2 \text{ km/h}$

a) A garden snail named Archie covered a 33 cm course in 2 minutes at the 1995 World Snail Racing Championships, held in England. What was Archie's average speed?

$$t = 2 \text{ min} = \frac{2}{60} \text{ h} = \frac{1}{30} \text{ h} \quad \text{Simplify: } \div 2$$

$$v = \frac{33 \text{ cm}}{\frac{1}{30} \text{ h}} = 33 \div \frac{1}{30} \text{ cm/h}$$

$$= 33 \times \frac{30}{1} \text{ cm/h} = \boxed{\text{cm/h}}$$

c) The Suzuki Hayabusa is the world's fastest motorbike. It can travel 100 km in 20 minutes. What is its average speed?

$$t = 20 \text{ min} =$$

$$v =$$

$$= \boxed{\text{km/h}}$$

e) A boat crosses a 3 km lake in 20 minutes. What is its average speed?

$$t = 20 \text{ min} =$$

$$v =$$

$$= \boxed{\text{km/h}}$$

b) The Gentoo penguin in the Antarctic Islands can swim 20 km in half an hour. What is its average speed?

$$t = 30 \text{ min} =$$

$$v =$$

$$= \boxed{\text{km/h}}$$

d) Some species of dolphins can swim 9 km in 10 minutes. What is their average speed in kilometres per hour?

$$t =$$

$$v =$$

$$= \boxed{\text{km/h}}$$

f) Johann jogs 12 km in one and a half hours. What is his average speed?

$$t =$$

$$v =$$

$$= \boxed{\text{km/h}}$$

Skill 10.6 Finding the distance travelled.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

$$\text{distance travelled } (d) = \text{speed } (v) \times \text{time taken } (t) \quad \text{OR} \quad d = vt$$

- Write the formula for distance travelled.
- Convert the given units into the required units if necessary. (see Maths Facts, page 455)
*Hints: Changing from smaller units into larger units, always divide by the conversion factor.
Changing from larger units into smaller units, always multiply by the conversion factor.*
- Substitute the values for average speed and time into the formula.
- Evaluate and simplify.

Q. The ride duration of the Disneyland monorail in California is approximately 15 minutes. What is the length of the ride if the train's average speed is 16 km/h?
[Give the answer in km.]

A. $d = vt$

$$\text{where } t = 15 \text{ min} = \frac{15}{60} \text{ h} = \frac{1}{4} \text{ h}$$

$$d = 16 \text{ km/h} \times \frac{1}{4} \text{ h} \quad \text{Substitute into the formula}$$

$$= \cancel{16}^4 \times \frac{1}{\cancel{4}_1} \text{ km} \quad \text{Simplify: } \div 4$$

$$= 4 \text{ km}$$

a) The Russian Alpha class nuclear-powered submarine has a maximum speed of 75 km/h, or 40 knots. At this speed, what distance can it cover in 12 hours?

$$d = 75 \text{ km/h} \times 12 \text{ h}$$

$$= 75 \times 12 \text{ km} = \boxed{\text{km}}$$

b) The Japanese Bullet train is the fastest scheduled train service in the world with an average speed of 260 km/h. At this speed, what distance can it cover in 3 hours?

$$d =$$

$$= \boxed{\text{km}}$$

c) The Nile flows at an average speed of around 7.5 km/h during inundation season. At this speed, what distance might a boat floating on the Nile travel in 6 hours?

$$d =$$

$$= \boxed{\text{km}}$$

d) The Eurostar trains operate from London to Brussels and run at an average speed of 140 km/h. If the trip takes two and half hours, what is the distance from London to Brussels?

$$d =$$

$$= \boxed{\text{km}}$$

e) A garden snail can travel at 0.012 m/s. At this speed, what distance can it cover in 10 minutes?

$$t = 10 \text{ min} = 10 \times 60 \text{ s} = 600 \text{ s}$$

Use $d = vt$

$$d = \boxed{\text{m}}$$

f) The F-16 Falcon fighter aircraft can fly at a speed of 2400 km per hour at sea level. At this speed, what distance can it cover in 20 min?

$$t =$$

$$d = \boxed{\text{km}}$$

Skill 10.7 Finding the time taken to travel a distance.

MM5.2 1 1 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

$$\text{time taken } (t) = \frac{\text{distance travelled } (d)}{\text{speed } (v)} \quad \text{OR} \quad t = \frac{d}{v}$$

- Write the formula for the time taken.
- Convert the given units into the required units if necessary. (see Maths Facts, page 455)
*Hints: Changing from smaller units into larger units, always divide by the conversion factor.
Changing from larger units into smaller units, always multiply by the conversion factor.*
- Substitute the values for the distance travelled and the average speed into the formula.
- Evaluate and simplify.

Q. Donghai Bridge China is the longest cross-sea bridge in the world, with a length of 32 km. How long will it take a car travelling at 80 km/h to cross the bridge? [Give the answer in hours.]

$$\begin{aligned} \text{A. } t &= \frac{d}{v} \\ &= \frac{32 \text{ km}}{80 \text{ km/h}} \quad \text{Substitute into the formula} \\ &= \frac{2 \cancel{32}}{5 \cancel{80}} \text{ h} \quad \text{Simplify: } \div 16 \\ &= 2 \div 5 \text{ h} \\ &= \mathbf{0.4 \text{ h}} \end{aligned}$$

a) The speed of long distance jogging for an average person is around 10 km/h. At this speed, how long will it take a person to run 15 km?

$$t = \frac{15 \text{ km}}{10 \text{ km/h}}$$

$$= 15.0 \div 10 \text{ h} = \boxed{} \text{ h}$$

b) A golf ball leaves the tee and flies at an average speed of 40 m/s till it reaches the 200 m mark. How long did it take the ball to fly this distance?

$$t =$$

$$= = \boxed{} \text{ s}$$

c) Top athletes can sprint at a speed around 10 m/s within a short distance. How long will it take an athlete to sprint 200 m?

$$t =$$

$$= = \boxed{} \text{ s}$$

d) The average speed of a space shuttle in orbit is 8000 m/s. At this speed, how long will it take a space shuttle to travel 1000 km?

$$t =$$

$$= = \boxed{} \text{ s}$$

e) The average walking speed for adults is 5 km/h. At this speed, how long will it take an adult to walk 7 km?

$$t =$$

$$= = \boxed{} \text{ min}$$

f) The Metro monorail in Sydney is 3.6 km long. At an average speed of 24 km/h, how long would it take the train to complete a loop?

$$t =$$

$$= = \boxed{} \text{ min}$$

Skill 10.8 Deciding if two ratios are in proportion.

MM5.2 11 22 33 44
MM6.1 11 22 33 44

- Write the two ratios as equal fractions side by side.
- Cross multiply the numerators and the denominators of the fractions.
- If the two products are equal, then the two ratios are equivalent (or form a proportion).

A proportion $a:b = c:d$ *2 equal ratios*

$$\frac{a}{b} \times \frac{c}{d} \quad \text{Cross product}$$

$$a \times d = b \times c$$

$$ad = bc$$

- Q.** 5 : 2 is in proportion with 25 : 10
True or false?

A. $\frac{5}{2} \times \frac{25}{10}$ *Cross multiply*

$$5 \times 10 = 2 \times 25$$

$$50 = 50$$

true

- a)** 5 : 6 is in proportion with 3 : 5
True or false?

$$\frac{5}{6} \times \frac{3}{5} \Rightarrow 5 \times 5 = 6 \times 3$$

$$25 = 18$$

false

- b)** 8 : 12 is in proportion with 6 : 8
True or false?

- c)** 2 : 12 is in proportion with 3 : 18
True or false?

- d)** 9 : 15 is in proportion with 30 : 50
True or false?

- e)** 12 : 15 is in proportion with 4 : 5
True or false?

- f)** 8 : 20 is in proportion with 20 : 50
True or false?

- g)** 15 : 150 is in proportion with 6 : 20
True or false?

- h)** 8 : 10 is in proportion with 20 : 25
True or false?

- i)** 5 : 6 is in proportion with 10 : 18
True or false?

- j)** 2 : 3 is in proportion with 3 : 5
True or false?

Skill 10.9 Finding the missing term in a proportion.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Write the proportion as two equal fractions.
- Cross multiply the numerators and the denominators of the fractions.
- Equate the products.
- Solve the equation to find the missing number (x).

A proportion $a:b = c:d$ (2 equal ratios)

$$\frac{a}{b} = \frac{c}{d}$$

$$a \times d = b \times c$$

$$ad = bc$$

Q. Complete the missing term in the proportion:

10 is to = 5 is to 25

A. $\frac{10}{x} = \frac{5}{25}$

$\frac{10}{x} = \frac{5}{25}$ (Cross multiply)

$$10 \times 25 = x \times 5$$

$$5x = 250$$

$\frac{5x}{5} = \frac{250}{5}$ (Simplify: $\div 5$)

$$x = 50$$

a) Complete the missing term in the proportion:

$4 : 6 = 16 : \text{24}$

$\frac{4}{6} = \frac{16}{x}$ (Cross multiply)

$$4x = 96$$

$\frac{4x}{4} = \frac{96}{4}$ (Simplify: $\div 4$)
 $x = 24$

b) Complete the missing term in the proportion:

$5 : \text{ } = 50 : 100$

c) Complete the missing term in the proportion:

20 is to 15 = 8 is to

d) Complete the missing term in the proportion:

$8 : \text{ } = 10 : 15$

e) Find the missing term in the proportion:

$\frac{4}{12} = \frac{y}{9}$

$4 \times 9 = 12 \times y$ (Cross multiply)

$$12y = 36$$

f) Find the missing term in the proportion:

$\frac{6}{x} = \frac{2}{3}$

$y =$

$x =$

Skill 10.10 Dividing a quantity into a given ratio.

MM5.2 11 22 3 4 4
MM6.1 11 22 3 3 4 4

- Find the total number of equal parts, by adding the numbers in the ratio.
- Calculate what fraction each share represents out of the total number of parts.
- Multiply this fraction by the original quantity.

Q. The ratio of cement to sand to gravel in a concrete mix is 1 : 2 : 3. How much sand is in a 24 kg concrete mix?

A. $equal\ parts = 1 + 2 + 3 = 6$
 $sand\ share = 2\ out\ of\ 6 = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$
 $sand\ in\ 24\ kg = \frac{1}{3}\ of\ 24\ kg$
 $= \frac{1}{3} \times \cancel{24}^8\ kg \leftarrow \text{Simplify: } \div 3$
 $= 8\ kg$

a) The 18-carat gold is used for jewellery in a ratio of 3 : 1 pure gold to other metals. How many grams of pure gold are needed for a 12 g necklace?

$equal\ parts = 3 + 1 = 4$
 $pure\ gold\ share = \frac{3}{4}$
 $pure\ gold\ in\ 12\ g = \frac{3}{4}\ of\ 12\ g$
 $= \frac{3}{4} \times \cancel{12}^3\ g \leftarrow \text{Simplify: } \div 4 = \boxed{}\ g$

c) The ratio of vowels to consonants in the English language is 5 : 21. How many vowels are likely to be in a 52 000 letter article?

$equal\ parts =$
 $vowels\ share =$
 $vowels\ in\ 52\ 000\ letters =$
 $= = \boxed{}$

e) The ratio of gold to silver to bronze medals won by Switzerland at the 2010 Vancouver Winter Olympics is 2 : 0 : 1. If Switzerland won 9 medals in total, how many gold medals did they win?

$= = \boxed{}$

b) The ratio of marriages to divorces in Australia in 2006 was 2 : 1. How many divorces would be likely in 1500 couples?

$equal\ parts =$
 $divorce\ share =$
 $divorces\ in\ 1500\ couples =$
 $= = \boxed{}$

d) The fuel mix for a chainsaw is 4 parts oil to 21 parts petrol. How much petrol is in a 1500 mL chainsaw tank?

$= = \boxed{}\ mL$

f) The combined monthly bill for the phone, mobile phone and internet is \$180. If the ratio of phone to mobile phone to internet costs is 5 : 6 : 4, how much is the phone cost?

$= = \boxed{}\ \$$

Skill 10.11 Solving proportions.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Write the proportion using words.
- Replace the words with numbers:
First the given ratio.
Then the ratio of the given quantity to the unknown quantity (x).
- Rewrite the proportion as two equal fractions.
- Cross multiply the numerators and the denominators of the fractions.
- Equate the products.
- Solve the equation to find x .

A proportion $a:b = c:d$ 2 equal ratios

$$\frac{a}{b} \times \frac{c}{d}$$

$$a \times d = b \times c$$

$$ad = bc$$

- Q.** The ratio of concentrated cordial to water is 1 : 4. How much water is needed to dilute 1.5 L of concentrated cordial?

A. *cordial : water = cordial : water*
 $1 : 4 = 1.5 : x$

$$\frac{1}{4} \times \frac{1.5}{x}$$

Cross multiply

$$1 \times x = 4 \times 1.5$$

$$x = \mathbf{6\ L}$$

- a)** A risotto recipe uses a ratio of 2 cups of rice to 6 cups of water. How many cups of water have to be added to 6 cups of rice?

$$\text{rice} : \text{water} = \text{rice} : \text{water}$$

$$2 : 6 = 6 : x$$

$$\frac{2}{6} \times \frac{6}{x}$$

Cross multiply

$$2x = 36$$

Simplify: $\div 2$ So $x =$

- b)** The fuel mix for a chainsaw is 4 parts oil to 21 parts petrol. How much petrol needs to be added to 240 mL of oil?

$$=$$

$$=$$

$$\text{So } x =$$
 mL

- c)** The ratio of bronze to silver medals won by England at the 2010 Commonwealth games is 3 : 4. If England won 45 bronze medals, how many silver medals did it win?

$$=$$

$$=$$

$$\text{So } x =$$

- d)** To make a 25% saline solution, 1 part of salt is used for every 3 parts of water. If you use 120 g of salt, how much water do you need to make the saline solution?

$$=$$

$$=$$

$$\text{So } x =$$
 g

- e)** To mix concrete, 2 buckets of sand are needed for every 3 buckets of gravel. How many buckets of gravel are needed for 10 buckets of sand?

$$\text{sand} : \text{gravel} =$$

$$=$$

$$\text{So } x =$$

- f)** The superjumbo jet Airbus A380 has a length of 73 m and a wingspan of 80 m. A model of this plane has a wingspan of 160 cm. How long is the model?

$$=$$

$$=$$

$$\text{So } x =$$
 cm

Skill 10.12 Finding other rates.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

$$\text{rate} = \frac{\text{amount}}{\text{time}}$$

Rate of change
(amount and time are given)

- Convert the given units to the required units.
(see Maths Facts, page 451)
- Divide the amount by the time taken.

Example: A 300 L bathtub can be filled in 10 minutes.

$$\text{Rate} = \frac{300 \text{ L}}{10 \text{ min}} = 30 \text{ L/min}$$

$$\text{amount} = \text{rate} \times \text{time}$$

Amount
(rate and time are given)

- Convert the given units to the required units.
(see Maths Facts, page 451)
- Multiply the rate by the time taken.

Example: Sam worked 7 hours and was paid at a rate of \$16/h.

$$\text{Amount (pay)} = 16 \times 7 = \$112$$

$$\text{time} = \frac{\text{amount}}{\text{rate}}$$

Time taken
(amount and rate are given)

- Convert the given units to the required units.
(see Maths Facts, page 451)
- Divide the amount by the rate.

Example: A Lexmark E232 prints 990 pages at a rate of 22 pages/min (ppm).

$$\text{Time} = \frac{990 \text{ p}}{22 \text{ ppm}} = 45 \text{ min}$$

- Q.** The average pulse for a new born baby is around 130 beats per minute. How many beats in 3 hours is this?

A. $\text{amount} = \text{rate} \times \text{time}$

$$\text{rate} = 130 \text{ beats/min}$$

$$\text{time} = 3 \text{ h} = 3 \times 60 \text{ min} = 180 \text{ min}$$

$$\text{amount} = 130 \text{ beats/min} \times 180 \text{ min} = 23\,400 \text{ beats}$$

- a)** The ruby throated hummingbird can beat its wings around 21 000 times in 5 minutes. What is its wing beat rate in beats per second?

$$\text{time} = 5 \times 60 \text{ s} = 300 \text{ s}$$

$$\text{rate} = \frac{21\,000 \text{ beats}}{300 \text{ s}} = \boxed{\text{beats/s}}$$

Simplify: $\div 100$ then $\div 3$

- b)** The heart of an unborn baby beats at around 9000 times in an hour. What is the heart rate of an unborn baby in beats per minute?

$$\text{time} = 1 \times 60 \text{ min} = 60 \text{ min}$$

$$\text{rate} = \frac{\text{amount}}{\text{time}} = \boxed{\text{beats/min}}$$

- c)** Find the time taken to print 875 sheets of paper, if a Lexmark T640 printer can print 35 pages per minute.

$$\text{time} = \frac{\text{amount}}{\text{rate}}$$

$$= \frac{875}{35} = \boxed{\text{min}}$$

- d)** An adult's air intake adds up to 7500 L per day. How many litres of air does an adult breathe in a week?

$$\text{amount} = \text{rate} \times \text{time}$$

$$= 7500 \times 7 = \boxed{\text{L}}$$

- e)** Victoria has an average population density of around 22 people per square kilometre. If Victoria has an area of approximately 228 000 km², what is its population?

$$\text{population} = \text{area} \times \text{density rate}$$

$$= 228\,000 \times 22 = \boxed{\text{people}}$$

- f)** Bangladesh is the most densely populated country, with around 1050 people per km² in 2007. If in 2007 Bangladesh had a population of 151 200 000, what is its area in km²?

$$\text{area} = \text{population} \div \text{density rate}$$

$$= \frac{151\,200\,000}{1050} = \boxed{\text{km}^2}$$

Skill 10.13 Completing equivalent rates.

- Write the given rate as a fraction.
- Show the respective units in the fraction.
- Convert the units to match to the new rate.
- Use the conversion factors for units of time, weight and length.
- Multiply by the conversion factor to change from larger units to smaller units.

Example: To change mm to cm
÷ by 10

- Divide by the conversion factor to change from smaller units to larger units.
- Example: To change cm to mm
× by 10

Q. $2 \text{ km/min} = \boxed{} \text{ km/h}$

A. 2 km per minute
 $= \frac{2 \text{ km}}{1 \text{ min}}$ *1 h = 60 min*
 $= \frac{2 \text{ km}}{\frac{1}{60} \text{ h}}$ *1 min = $\frac{1}{60}$ h*
 $= 2 \times \frac{60 \text{ km}}{1 \text{ h}}$
 $= \mathbf{120 \text{ km/h}}$

a) $144 \text{ L/h} = \boxed{} \text{ mL/h}$

144 L per hour *Convert L to mL*
 $= \frac{144 \text{ L}}{1 \text{ h}} = \frac{144000 \text{ mL}}{1 \text{ h}} = 144000 \text{ mL/h}$

b) $10 \text{ km/h} = \boxed{} \text{ m/h}$

$= = = $

c) $3000 \text{ g/min} = \boxed{} \text{ kg/min}$

$= = = $

d) $\$5/\text{kg} = \boxed{} \text{ cents/kg}$

$= = = $

e) $60 \text{ mm/s} = \boxed{} \text{ mm/min}$

$= = = $

f) $5 \text{ answers/min} = \boxed{} \text{ answers/h}$

$= = = $

g) $2000 \text{ mm/s} = \boxed{} \text{ m/s}$

$= = = $

h) $480 \text{ beats/h} = \boxed{} \text{ beats/min}$

$= = = $

Skill 10.14 Identifying direct proportion in real life situations.

MM5.2 11 22 33 44
MM6.1 11 22 33 44

- Write the rates of the two quantities as fractions.
The rate of the units (e.g. km per hour = km/h).
The first rate of the given quantities.
The second rate containing the unknown quantity (x).
- Write the proportion as two equal fractions.
- Cross multiply the numerators and the denominators of the fractions.
- Equate the products.
- Solve the equation to find x .

A proportion $a:b = c:d$ (2 equal ratios)

$$\frac{a}{b} = \frac{c}{d}$$

$$a \times d = b \times c$$

$$ad = bc$$

Q. Lin can write 7 pages in 24 minutes. Assuming her writing rate is directly proportional, how many pages can she write in 120 minutes?

A. rate of units: $\frac{\text{pages}}{\text{minute}}$

1st rate: $\frac{7}{24}$

2nd rate: $\frac{x}{120}$

Cross multiply

$$\frac{7}{24} = \frac{x}{120}$$

$$7 \times 120 = 24 \times x$$

$$24x = 840$$

$$x = 35$$

Let x be the number of pages Lin can write in 120 minutes.

Write the proportion. Cross multiply.

Solve for x .

a) I can buy eight pens for \$4.50. How many pens can I buy for \$18?

rate of units: $\frac{\text{pens}}{\$}$ 1st rate: $\frac{8}{4.50}$

2nd rate: $\frac{x}{18} \Rightarrow \frac{8}{4.50} = \frac{x}{18}$

$\Rightarrow 8 \times 18 = 4.5 \times x$ $x =$

b) If 6 m of fabric cost \$75, what is the cost of 4 m of the same fabric?

rate of units: $\frac{m}{\$}$ 1st rate: $\frac{6}{75}$

2nd rate: \Rightarrow

\Rightarrow $x =$

c) It took Jon 40 minutes to run 6 laps of the oval. Assuming his running rate is directly proportional, how long it would take him to run 9 laps?

rate of units: 1st rate:

2nd rate: \Rightarrow

\Rightarrow $x =$ min

d) Paper costs \$4.50 for 500 sheets. What is the cost for 200 sheets?

rate of units: 1st rate:

2nd rate: \Rightarrow

\Rightarrow $x =$

e) Fred's respiration rate is 62 breaths in 4 minutes. At this rate, how many breaths will he take in 20 minutes?

rate of units: 1st rate:

2nd rate: \Rightarrow

\Rightarrow $x =$

f) Tony eats 5 fruits in 2 days. Assuming his eating rate is directly proportional, how many fruits does Tony eat in 16 days?

rate of units: 1st rate:

2nd rate: \Rightarrow

\Rightarrow $x =$

$$\text{speed} = \frac{\text{distance}}{\text{time}} \quad \text{OR} \quad v = \frac{d}{t}$$

Convert km/h to m/s

- Write the speed as distance over time.
- Change km to m by multiplying by 1000.
- Change h to s by multiplying by $60 \times 60 = 3600$.
- Evaluate and simplify.

Convert m/s to km/h

- Write the speed as distance over time.
- Change m to km by dividing by 1000.
- Change s to h by dividing by 60 to get to mins and by 60 again to get to hours.
- Evaluate and simplify.

Q. The fastest combat jet is the former Soviet MIG 25, which reached a speed of 940 m/s. What is the speed in km/h?

A.

$$v = \frac{d}{t}$$

$$= \frac{940 \text{ m}}{1 \text{ s}}$$

change m to km: $\div 1000$

$$= \frac{940 \div 1000}{1 \div 3600}$$

change s to h: $\div 3600$

$$= \frac{940}{1000} \div \frac{1}{3600} \text{ km/h}$$

fraction line means division

$$= \frac{940}{1000} \times \frac{3600}{1} \text{ km/h}$$

Simplify: $\div 100$ then $\div 10$

$$= \frac{940}{10000} \times 36000 \text{ km/h}$$

$$= 3384 \text{ km/h}$$

a) Shoaib Akhtar, Pakistan, has bowled the fastest ball in Test cricket with a speed of nearly 162 km/h. How many m/s is this?

$$v = \frac{162 \text{ km}}{1 \text{ h}}$$

$$= \frac{162 \times 1000 \text{ m}}{1 \times 3600 \text{ s}} = \frac{270}{6} = \boxed{} \text{ m/s}$$

Simplify: $\div 100$ then $\div 6$

b) The fastest propelled aircraft is the former Soviet Tupolev Tu-95/142, which reached a speed of 250 m/s. How many km/h is this?

$$v = 250 \text{ m/s} = \frac{250 \div 1000 \text{ km}}{1 \div 3600 \text{ h}}$$

$$= = \boxed{} \text{ km/h}$$

c) The average speed of a space shuttle in orbit is 28 800 km/h. How many m/s is this?

$$v =$$

$$= = \boxed{} \text{ m/s}$$

d) The earth is moving around the sun at a speed of about 30 000 m/s. How many km/h is this?

$$v =$$

$$= = \boxed{} \text{ km/h}$$

e) The fastest electric vehicle reached 396 km/h in Utah, USA, in 1999. What is this speed in m/s?

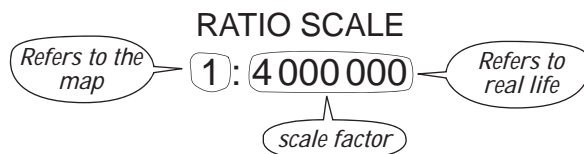
$$v =$$

$$= = \boxed{} \text{ m/s}$$

f) While crossing the Eurotunnel the Eurostar train can reach speeds of up to 85 m/s. How many km/h is this?

$$v =$$

$$= = \boxed{} \text{ km/h}$$



*Hint: 1 unit on the map represents 4 000 000 of the same units in real life.
The real distance is 4 000 000 times bigger than the map distance.*

Finding the real life distance (ratio scale and distance on the map are given)

- Multiply the distance on the map (usually in cm) by the scale factor.
- Convert the result from cm to km, by dividing by 100 000.

Finding the distance on the map (ratio scale and real life distance are given)

- Convert the real distance from km to cm by multiplying by 100 000.
- Divide the real distance in cm by the scale factor.

Finding the scale factor (real life distance and distance on the map are given)

- Convert the real distance from km to cm by multiplying by 100 000.
- Divide the real distance in cm by the map distance in cm.

Q. On a map the scale is 1 : 10 000 000. What is the distance on the map between Brisbane and Adelaide, if they are 2000 km apart?

A. *real life distance = 2000 km*
scale factor = 10 000 000
map distance = ?

2000 km = 2000 × 100 000 cm =
= 200 000 000 cm

200 000 000 cm ÷ 10 000 000 =
= 20 cm

Convert km to cm

Cross off respective 0's

a) On a map the scale ratio is 1 : 20 000 000. What is the real distance between Tokyo and Geneva, if they are 80 cm apart on the map?

real distance = 80 cm × 20 000 000

= 1 600 000 000 cm

1 600 000 000 cm ÷ 100 000 = km

Cross off respective 0's

b) On a map the scale ratio is 1 : 30 000 000. What is the real distance between Sydney and London, if they are 70 cm apart on the map?

real distance =

=

= km

c) On a map the scale ratio is 1 : 25 000 000. What is the real distance between Broome and Melbourne, if they are 20 cm apart on the map?

real distance =

=

= km

d) On a map the scale ratio is 1 : 3 000 000. What is the real distance between Sydney and Canberra, if they are 10 cm apart on the map?

real distance =

=

= km

Skill 10.16 Working with ratio scales (2).

- e)** The Empire State Building is 380 m high. If you are drawing a model using a scale of 1 : 2000, how high is the model?

real height = 380 m = 38 000 cm

model height = 38 000 cm ÷ 2000

= cm

- f)** Taipei 101 is the world's tallest building, with a height of 500 m. If you are building a model using a scale ratio of 1 : 1000, how tall is the model?

real height =

model height =

= cm

- g)** On a map the scale ratio is 1 : 10 000 000. What is the distance on the map between Sydney and Singapore, if the real life distance is 4000 km?

real distance =

map distance =

= cm

- h)** On a town plan the scale ratio is 1 : 100 000. What is the distance on the map between the airport and the post office, if they are 15 km apart?

real distance =

map distance =

= cm

- i)** On a map the distance between Perth and Cairns is 50 cm. What is the scale factor of the map, if the real life distance is 6000 km?

real distance = 6000 × 100 000 cm

= 600 000 000 cm

scale factor = 600 000 000 cm ÷ 50 cm

= 12 000 000 =

- j)** The Walt Disney studios' main theatre screen dimensions are 14 m wide by 5 m high. If you make a model 2.5 cm high, what is the scale factor used to build the model?

real height =

=

scale factor =

=

- k)** On a map the distance between Darwin and Perth is 20 cm. What is the scale factor of the map, if the real life distance is 4000 km?

real distance =

=

scale factor =

=

- l)** Petronas Towers in Kuala Lumpur are 450 m tall. A model has a height of 30 cm. What scale factor was used to build the model?

real height =

=

scale factor =

=

Skill 10.17 Comparing rates.

MM5.2 11 22 33 44
MM6.1 11 22 33 44

- Write the rates as fractions.
- Bring the rates to the same unit. (see Maths Facts, page 455)
Hint: It is easier to change from larger units into smaller units, because you have to multiply by the conversion factor.
- Evaluate and simplify.
- Compare the numbers.

Q. Which density is lower?

- A) 670 kg/m³ (gasoline)
B) 1.025 g/cm³ (seawater)

A. kg/m³ to g/cm³: kg to g ⇒ × 1000

$$m^3 \text{ to } cm^3 \Rightarrow \times 1\,000\,000$$

$$A) \text{ density} = \frac{670 \text{ kg}}{1 \text{ m}^3}$$

$$= \frac{670 \times 1\,000\,000 \text{ g}}{1 \times 1\,000\,000\,000 \text{ cm}^3}$$

*cross off
respective 0's*

$$= \frac{67}{100} \text{ g/cm}^3$$

$$= 0.67 \text{ g/cm}^3$$

$$B) \text{ density} = 1.025 \text{ g/cm}^3$$

0.67 < 1.025, so the answer is **A**

a) Which heartbeat rate is higher?

- A) 70 beats/min (adult)
B) 2 beats/s (baby)

$$B) \text{ rate} = \frac{2 \text{ beats}}{1 \text{ s}} = \frac{2 \text{ beats}}{\frac{1}{60} \text{ min}}$$

$$= 2 \div \frac{1}{60} = 2 \times \frac{60}{1} = 120 \text{ beats/min}$$

$$120 \text{ beats/min} > 70 \text{ beats/min}$$

B

b) Which heartbeat rate is lower?

- A) 1.5 beats/s (child)
B) 70 beats/min (adult)

$$A) \text{ rate} =$$

c) Which birth rate is lower?

- A) 12.1 per 1000 people (Australia)
B) 82 per 10000 people (Germany)

$$A) \text{ rate} = \frac{12.1}{1000} =$$

d) Which birth rate is higher?

- A) 14.1 per 1000 people (USA)
B) 94 per 10000 people (Japan)

e) Which density is lower?

- A) 800 kg/m³ (petrol)
B) 1 g/cm³ (water)

f) Which density is higher?

- A) 2.2 g/cm³ (graphite)
B) 1300 kg/m³ (PVC)