

13. [Exploring Number]

Skill 13.1 Using 'order of operations' involving a mix of (), ×, +, + or -

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Simplify inside the brackets.
- Multiply (×) and/or divide (÷) in order from left to right.
- Add (+) and/or subtract (-) in order from left to right.

Q. $18 \div (9 - 3) + 2 =$

A. $18 \div (9 - 3) + 2 =$
 $= 18 \div 6 + 2$ *subtract inside the brackets*
 $= 3 + 2$ *division before addition*
 $= 5$

a) $6 + 12 \div 4 \times 3 =$
 $= 6 + 3 \times 3$
 $= 6 + 9$ =

b) $6 \times 15 - 8 \times 3 =$
 $=$
 $=$ =

c) $5 + 12 \div 6 \times 3 =$
 $=$
 $=$ =

d) $3 \times (5 - 3) \times 8 =$
 $=$
 $=$ =

e) $(15 + 8) - (7 + 6) =$
 $=$
 $=$ =

f) $120 \div 5 - 6 \times 3 =$
 $=$
 $=$ =

g) $22 - 8 - (11 - 4) =$
 $=$
 $=$ =

h) $20 - (15 - 9) + 6 =$
 $=$
 $=$ =

i) $6 \times (14 + 7) =$
 $=$
 $=$ =

j) $8 \times 5 \div (7 - 3) =$
 $=$
 $=$ =

k) $4 + (9 - 4) \times 7 =$
 $=$
 $=$ =

l) $36 - 2 \times (12 + 5) =$
 $=$
 $=$ =

m) $144 \div 6 - 4 \times 5 + 18 \div 3 =$
 $=$
 $=$ =

n) $40 - 3 \times (13 + 5) \div 3 + 12 =$
 $=$
 $=$ =

Skill 13.2 Using 'order of operations' involving powers and (), ×, ÷, + or -

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Simplify inside the brackets.
- Simplify the power.
- Always multiply (×) and/or divide (÷) in order from left to right.
- Always add (+) and/or subtract (-) in order from left to right.

Q. $(6 + 2 \times 5)^2 =$

A. $(6 + 2 \times 5)^2 =$
 $= (6 + 10)^2$ *multiply within brackets first*
 $= 16^2$ *add inside the brackets*
 $= 256$

a) $(3 \times 5)^2 =$
 $= 15^2$ =

b) $(2 \times 7)^2 =$
 $=$ =

c) $(5 + 5 \times 3)^2 =$
 $=$
 $=$ =

d) $(2 \times 4 + 6)^2 =$
 $=$
 $=$ =

e) $(2 + 8)^2 \div 4 =$
 $=$
 $=$ =

f) $(7 + 5)^2 \div 8 =$
 $=$
 $=$ =

g) $5 + (12 - 6)^2 =$
 $=$
 $=$ =

h) $8 + (13 - 8)^2 =$
 $=$
 $=$ =

i) $(4 \times 2 + 2)^2 =$
 $=$
 $=$ =

j) $(3 \times 4 + 8)^2 =$
 $=$
 $=$ =

k) $3 + (1 + 8)^2 =$
 $=$
 $=$ =

l) $6 + (7 + 1)^2 =$
 $=$
 $=$ =

m) $(10 - 1)^2 \div (30 - 3) =$
 $=$
 $=$ =

n) $(10 - 3)^2 \div (12 - 5) =$
 $=$
 $=$ =

Rounding terminating decimals to a given place

- Circle the digit to the right of the requested place.
- If this digit is: 0, 1, 2, 3 or 4 (< 5) - **round down** - keep the digit in the requested place the same.
5, 6, 7, 8 or 9 (≥ 5) - **round up** - add 1 to the digit in the requested place.

Rounding recurring decimals to a given place

- Write the first 4 digits after the decimal point. (see skill 7.14, page 84)
- Apply the procedure described above for terminating decimals.

Q. Round $0.4\dot{6}$ to 3 decimal places.

A. $0.4\dot{6} = 0.4666\dots$

$0.466\textcircled{6}\dots$

≈ 0.467

a) Round 0.13 to 1 decimal place.

$0.1\textcircled{3}$ \approx

b) Round 7.89 to 1 decimal place.

..... \approx

c) Round 12.45 to 1 decimal place.

..... \approx

d) Round 31.5841 to 2 decimal places.

$31.58\textcircled{4}1$ \approx

e) Round 24.793 to 2 decimal places.

..... \approx

f) Round 4.231 to 2 decimal places.

..... \approx

g) Round 3.859 to 1 decimal place.

..... \approx

h) Round 50.296 to 2 decimal places.

..... \approx

i) Round $4.\dot{7}$ to 2 decimal places.

$4.\dot{7} = 4.77\textcircled{7}7\dots$ \approx

j) Round $3.4\dot{2}$ to 2 decimal places.

..... \approx

k) Round $0.\dot{6}$ to 2 decimal places.

..... \approx

l) Round $1.7\dot{3}$ to 3 decimal places.

..... \approx

m) Round $4.2\dot{8}$ to 3 decimal places.

..... \approx

n) Round $0.1\dot{6}$ to 3 decimal places.

..... \approx

Skill 13.4 Writing rational approximations of simple irrational numbers.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Circle the digit to the right of the requested place.
- If this digit is: 0, 1, 2, 3 or 4 (< 5) - **round down** - keep the digit in the requested place the same.

5, 6, 7, 8 or 9 (≥ 5) - **round up** - add 1 to the digit in the requested place.

Hint: To write a decimal number correct to two decimal places is the same thing as rounding off to the nearest hundredth.

Irrational number $\sqrt{2} \approx 1.41421356$ Rational approximation

Q. $\cos 45^\circ \approx 0.70711$

Write the rational approximation of $\cos 45^\circ$ correct to two decimal places.

A. 0.70711

circle the third digit

≈ 0.71

$7 \geq 5$
round up by adding 1 to 0

a) $\sqrt{12} \approx 3.46410162$

Write the rational approximation of $\sqrt{12}$ correct to two decimal places.

$3.46410162 \approx \boxed{3.46}$

*4 < 5
round down by keeping 6*

b) $\sqrt{20} \approx 4.47213595$

Write the rational approximation of $\sqrt{20}$ correct to two decimal places.

$\dots \approx \boxed{}$

c) $\sqrt{24} \approx 4.89897949$

Write the rational approximation of $\sqrt{24}$ correct to two decimal places.

$\dots \approx \boxed{}$

d) $\sqrt{30} \approx 5.47722558$

Write the rational approximation of $\sqrt{30}$ correct to two decimal places.

$\dots \approx \boxed{}$

e) $\pi \approx 3.14159265$

Write the rational approximation of π correct to three decimal places.

$\dots \approx \boxed{}$

f) $\varphi \approx 1.61803398$ (the golden ratio)

Write the rational approximation of φ correct to three decimal places.

$\dots \approx \boxed{}$

g) $\sin 15^\circ \approx 0.25882$

Write the rational approximation of $\sin 15^\circ$ correct to three decimal places.

$\dots \approx \boxed{}$

h) $\tan 60^\circ \approx 1.73205$

Write the rational approximation of $\tan 60^\circ$ correct to three decimal places.

$\dots \approx \boxed{}$

i) $e \approx 2.71828182$ (Euler's number)

Write the rational approximation of e correct to two decimal places.

$\dots \approx \boxed{}$

j) $\sqrt{10} \approx 3.16227766$

Write the rational approximation of $\sqrt{10}$ correct to three decimal places.

$\dots \approx \boxed{}$

Skill 13.6 Writing a number in scientific notation as a basic numeral.

MM5.2 11 2 2 3 4 4
MM6.1 11 2 2 3 3 4 4

$$2.43 \times 10^5 = 243\,000$$

scientific notation

basic numeral

Product of: Number ≥ 1 and < 10
Power of 10 with positive index

Very large

$$8.02 \times 10^{-4} = 0.000802$$

scientific notation

basic numeral

Product of: Number ≥ 1 and < 10
Power of 10 with negative index

Very small

If the power of 10 is **positive**:

- Move the decimal point to the right as many places as the power of 10.
- Add zeros as place holders if necessary.

Example: $3.1 = 3.1000$

Hint: By convention $37 = 37. = 37.0$

If the power of 10 is **negative**:

- Move the decimal point to the left as many places as the power of 10.
- Add zeros as place holders if necessary.
- If the result is less than 1, write a zero in the units place.

Hint: By convention 0.37 not $.37$

Q. Write 3.5×10^{-4} m, the diameter of optical fibre, as a basic numeral.

A. $3.5 \times 10^{-4} = 00003.5 \times 10^{-4} = 0.00035$

index = -4
move decimal point 4 places left
add zeros as place holders

a) 6.2×10^5 is the scientific notation for:
A) 6200 B) 620 000 C) 6.20000

$$6.2 \times 10^5 =$$

index = +5

$$= 620\,000.$$

5 places right

B

b) 4.12×10^6 is the scientific notation for:
A) 4 120 000 B) 412 000 C) 4.120000

$$=$$

c) 2.15×10^3 is the scientific notation for:
A) 2.15000 B) 215 000 C) 2150

$$=$$

d) 1.8×10^7 is the scientific notation for:
A) 1 800 000 B) 18 000 000 C) 180 000

$$=$$

e) Earth's atmosphere extends upward for 9.65×10^5 m. Write this as a basic numeral.

$$=$$

f) Write 1.4×10^9 , China's population in 2010, as a basic numeral.

$$=$$

g) The size of a red blood cell, 8.0×10^{-3} mm, is scientific notation for:
A) 0.0008 B) 8000 C) 0.008

$$=$$

h) The size of a virus, 2.5×10^{-5} mm, is scientific notation for:
A) 0.00025 B) 0.000025 C) 250 000

$$=$$

i) Write 2.5×10^{-11} m, the radius of a hydrogen atom, as a decimal number.

$$=$$

j) Write 5×10^{-7} m, the size of a speck of dust, as a decimal number.

$$=$$

Skill 13.7 Using 'order of operations' involving negative numbers.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Simplify within the brackets.
- Simplify the power.
- Always multiply (\times) and/or divide (\div) in order from left to right.
- Always add ($+$) and/or subtract ($-$) in order from left to right.
- Use the addition, subtraction, multiplication and division rules of negative numbers.

Q. $-48 \div (-8 - 4) + 23 =$

A. $-48 \div (-8 - 4) + 23 =$
 $= -48 \div (-12) + 23$ *subtract inside the brackets*
 $= 4 + 23$ *division before addition*
 $= 27$

a) $-6 \times (24 + 6) =$

$= -6 \times 30 = \boxed{-180}$

b) $-4 \times (7 + 9) =$

$= \dots\dots\dots = \boxed{}$

c) $(10 - 2) \div (3 - 7) =$

$= \dots\dots\dots = \boxed{}$

d) $(11 - 3) \div (1 - 9) =$

$= \dots\dots\dots = \boxed{}$

e) $(6 - 10) \times (-4 - 8) =$

$= \dots\dots\dots = \boxed{}$

f) $(-5 - 4) \times (2 - 5) =$

$= \dots\dots\dots = \boxed{}$

g) $(-9 - 2) \times (12 - 7) =$

$= \dots\dots\dots = \boxed{}$

h) $(-8 + 3) \times (6 - 12) =$

$= \dots\dots\dots = \boxed{}$

i) $8 \times 12 \div (2 - 6) =$

$= \dots\dots\dots$
 $= \dots\dots\dots = \boxed{}$

j) $5 \times 14 \div (3 - 10) =$

$= \dots\dots\dots$
 $= \dots\dots\dots = \boxed{}$

k) $-5 - 4 \times (11 - 9) =$

$= \dots\dots\dots$
 $= \dots\dots\dots = \boxed{}$

l) $-7 + 2 \times (15 - 4) =$

$= \dots\dots\dots$
 $= \dots\dots\dots = \boxed{}$

m) $-24 \div (-3 - 3) + 17 =$

$= \dots\dots\dots$
 $= \dots\dots\dots = \boxed{}$

n) $-12 \times 5 - 45 \div 9 =$

$= \dots\dots\dots$
 $= \dots\dots\dots = \boxed{}$

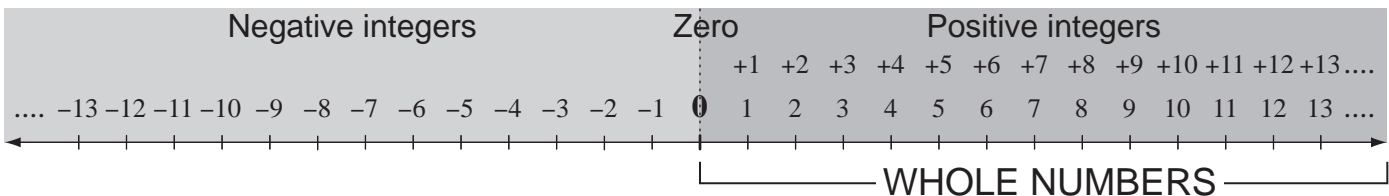
o) $-18 + 4 \times (7 + 8) \div 10 - 2 =$

$= \dots\dots\dots$
 $= \dots\dots\dots = \boxed{}$

p) $5 - 3 \times (6 + 2) \div 2 - 8 =$

$= \dots\dots\dots$
 $= \dots\dots\dots = \boxed{}$

INTEGERS



Whole Numbers

Negative integers, fractions, terminating decimals, recurring decimals and infinite non-recurring decimals are **not** whole numbers apart from these 2 situations:

- 1) Any positive fraction whose numerator is divisible by the denominator. $\frac{12}{4} = 3$
- 2) Any positive decimal with only zeros after the decimal point. $8.00 = 8$
- 3) Any square root of a perfect square. $\sqrt{36} = 6$

Integers

Fractions, terminating decimals, recurring decimals and infinite non-recurring decimals are **not** integers, apart from these 3 situations:

- 1) Any fraction whose numerator is divisible by the denominator. $-\frac{5}{1} = -5$
- 2) Any decimal with only zeros after the decimal point. $-3.00 = -3$
- 3) Any square root of a perfect square. $-\sqrt{16} = -4$

Q. Choose the whole numbers from this list:

$-7, \frac{8}{2}, -\frac{1}{3}, 0, -3.6, 50$

A. -7 is negative, so not a whole number

$\frac{8}{2} = 8 \div 2 = 4$ is a whole number

$-\frac{1}{3}$ is a fraction, so not a whole number

-3.6 is a decimal, so not a whole number

So $\frac{8}{2}, 0, 50$ are whole numbers.

a) Choose the whole numbers from this list:

$7.43, (89), -5, 3\frac{1}{5}, (14), 0.6$

b) Choose the whole numbers from this list:

$567, 0.73, -4, \frac{3}{10}, 12, 0$

c) Choose the whole numbers from this list:

$1.4142, 18, -5.\dot{9}, \frac{4}{11}, -5, 143$

d) Choose the whole numbers from this list:

$-25, 0.6666\dots, 34, \frac{5}{7}, -1, 8.93567, 2$

e) Choose the integers from this list:

$-3.5, 11, 2.\dot{1}4, -1, 3\frac{2}{7}, 2$

f) Choose the integers from this list:

$3.14, \frac{16}{4}, -3, -0.\dot{7}2, \sqrt{25}$

g) Choose the integers from this list:

$-75, 2.23607, -\frac{8}{2}, \sqrt{90}, 10.00$

h) Choose the integers from this list:

$-\sqrt{4}, \frac{\pi}{4}, 0.5252, 18, 0$

Skill 13.9 Recognising rational and irrational numbers.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

A number is **rational** if:

- It can be written as a fraction of 2 integers.

Hint: All integers are rational numbers.

$$-2, 700, \sqrt{16}, \frac{5}{1}, \frac{25}{5}$$

All terminating decimals are rational.

$$2.16, -5.753469$$

All recurring decimals are rational.

$$0.57575757... = 0.5\dot{7}$$

A number is **irrational** (not rational) if:

- It can be written as a decimal, but not as a fraction.
- It has infinite non-recurring digits after the decimal point.

Example: 2.52849302953...

Hint: Square roots of prime numbers and rational numbers that are not perfect squares are irrational numbers.

$$\sqrt{5}, \sqrt{18}$$

Q. Which numbers are rational?

- A) $-\sqrt{\frac{3}{5}}$
 B) 0.999...
 C) π
 D) $\frac{11}{2}$

A. $-\sqrt{\frac{3}{5}}$ is irrational, because $\frac{3}{5}$ is not a perfect square.
 0.999... is rational, because it is a recurring decimal.
 π is irrational, because it has infinite non-recurring digits after the decimal point ($\pi \approx 3.1415926535...$)
 $\frac{11}{2}$ is rational, because it is a fraction.
 So **B and D** are rational.

a) Is $\sqrt{7}$ a rational or an irrational number?

irrational

b) Is $4.12\dot{6}\dot{3}$ a rational or an irrational number?

c) Is $\frac{48}{25}$ a rational or irrational number?

d) Is 1.72430982... a rational or irrational number?

e) Is -60 a whole number, an integer or an irrational number?

f) Is 2.676767... a whole number, an integer or a rational number?

g) Is $-\frac{7}{2}$ a whole number, an integer or a rational number?

h) Is -12 000 a whole number, an integer or an irrational number?

i) Which is **not** a rational number?

- A) -6 B) 0.18952
 C) π D) $-\sqrt{9}$

j) Which is **not** a rational number?

- A) $-0.3\dot{1}$ B) $\sqrt{3}$
 C) 2.135135135... D) $\frac{11}{49}$

k) Which is an irrational number?

- A) 3 B) -2.5
 C) $\sqrt{4}$ D) $-\sqrt{2}$

l) Which is an irrational number?

- A) $2.\dot{6}$ B) 6.15
 C) $\sqrt{7}$ D) $5\frac{3}{10}$

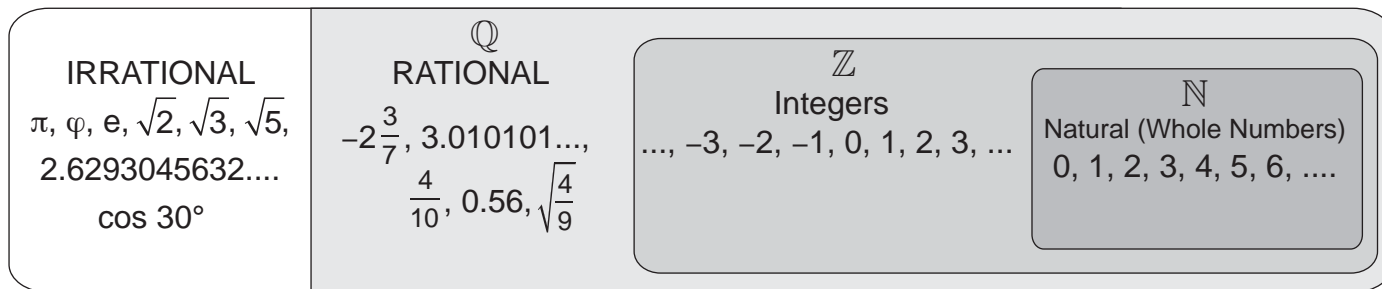
m) Which numbers are rational?

- A) $\sqrt{8}$ B) $6.5\dot{9}$
 C) -4.131133111333... D) 3.161616...

n) Choose the rational numbers from this list:

$$-2012, \pi, 0, \frac{28}{11}, -\sqrt{6}$$

\mathbb{R} REAL NUMBERS



*Hint: Rational numbers include integers, terminating decimals and recurring decimals.
Irrational numbers include infinite non-recurring decimals.*

\mathbb{N} included in \mathbb{Z} , \mathbb{Z} included in \mathbb{Q} , \mathbb{Q} included in \mathbb{R}

Irrational number included in \mathbb{R}

- | | |
|--|--|
| <p>Q. Which classification describes $-\sqrt{81}$?</p> <p>A) integer and irrational
 B) rational and real
 C) irrational and rational
 D) real and natural</p> | <p>A. $-\sqrt{81} = -9$</p> <p style="text-align: right;"> <i>integer</i> ✓
 <i>rational</i> ✓
 <i>real number</i> ✓
 <i>natural</i> ✗
 <i>irrational</i> ✗ </p> |
|--|--|

So B is the correct description.

a) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
4.327	false	true	false	true

b) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
-500				

c) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
π				

d) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
$\frac{3}{14}$				

e) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
$\sqrt{26}$				

f) Use true and false to complete this table:

	Integer	Rational	Irrational	Real
$\frac{36}{9}$				

g) Which classification describes 0.65291...?

- A) integer and rational
 B) rational and real
 C) integer and irrational
 D) irrational and real

h) Which classification describes $-\sqrt{49}$?

- A) integer and rational
 B) irrational and real
 C) integer and irrational
 D) rational and irrational

i) Which classification describes 0.153846?

- A) integer and irrational
 B) irrational and real
 C) integer and rational
 D) rational and real

j) Which classification describes $\frac{257}{43}$?

- A) integer and rational
 B) irrational and real
 C) rational and real
 D) rational and irrational

Skill 13.11 Comparing and ordering rational and irrational numbers.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Express the numbers as decimals.
- Round the decimal numbers to one or two decimal places, as needed in the question.

Examples: $\pi = 3.1415926... \approx 3.14$ correct to 2 decimal places

$$\frac{5}{3} = 1.6666... \approx 1.66 \text{ correct to 2 decimal places}$$

$$\sqrt{3} = 1.732050808... \approx 1.73 \text{ correct to 2 decimal places}$$

- Find the perfect squares greater than ($>$) and less than ($<$) the number under the square root.
- Compare and order the terminating decimals.

Q. Place in ascending order:

$$\sqrt{10}, \frac{10}{3}, 3.21, \pi, \sqrt{12}$$

express numbers as decimals

A. $\sqrt{10} = 3.16227766... \approx 3.16$

$$\frac{10}{3} = 3.33333... \approx 3.33 \text{ (round to 2 decimal places)}$$

$$\pi = 3.1415... \approx 3.14$$

$$\sqrt{12} = 3.46410161... \approx 3.46$$

$$\Rightarrow 3.14 < 3.16 < 3.21 < 3.33 < 3.46$$

$$\text{OR } \pi < \sqrt{10} < 3.21 < \frac{10}{3} < \sqrt{12}$$

The answer is $\pi, \sqrt{10}, 3.21, \frac{10}{3}, \sqrt{12}$

a) Which number is greater?

$$\sqrt{21} \text{ or } 5$$

$$16 < 21 < 25 \Rightarrow 4 < \sqrt{21} < 5 \Rightarrow \boxed{5}$$

b) Which number is greater?

$$\sqrt{72} \text{ or } 8$$

$$\dots \Rightarrow \boxed{}$$

c) Which number is smaller?

$$1.41 \text{ or } \sqrt{2}$$

$$\dots \Rightarrow \boxed{}$$

d) Which number is greater?

$$\sqrt{8} \text{ or } 3$$

$$\dots \Rightarrow \boxed{}$$

e) Which number is greater?

$$\pi \text{ or } \sqrt{9}$$

$$\dots \Rightarrow \boxed{}$$

f) Which number is smaller?

$$\frac{5}{2} \text{ or } \sqrt{6}$$

$$\dots \Rightarrow \boxed{}$$

g) Which number is greater?

$$\sqrt{18} \text{ or } 4$$

$$\dots \Rightarrow \boxed{}$$

h) Which number is smaller?

$$\sqrt{25} \text{ or } 5.1$$

$$\dots \Rightarrow \boxed{}$$

i) Place in ascending order:

$$\sqrt{5}, \frac{7}{3}, 2.\dot{2}, \frac{5}{2}, 2.4$$

$$\dots \Rightarrow \boxed{}$$

j) Place in ascending order:

$$\sqrt{8}, \frac{8}{3}, \frac{10}{4}, 2.76, \sqrt{7}$$

$$\dots \Rightarrow \boxed{}$$

