

# 21. [Coordinate Geometry]

## Skill 21.1 Completing a table of values for a linear rule.

MM5.2 1 2 2 3 3 4 4  
MM6.1 1 1 2 2 3 3 4 4

- Substitute the variable  $x$  with the given values.
- Calculate the value of  $y$ .
- Use the order of operations rules: Multiply ( $\times$ ) and/or divide ( $\div$ ) from left to right.  
Add ( $+$ ) and/or subtract ( $-$ ) from left to right.
- Use the sign rules. (see skill 9.1, page 93)

**Q.** Complete this table of values for the linear rule  $y = -4x + 2$

$x$	$y = -4x + 2$	$y$	$(x, y)$
-1	$y = -4 \times (-1) + 2$	6	$(-1, 6)$
0			
1			

**A.**  $y = -4x + 2$   $4x = 4 \times x$   
 $y = -4 \times x + 2$  Substitute  $x = 0$   
 $y = -4 \times 0 + 2$  Multiply  $-4$  by  $0$   
 $y = 0 + 2$  Add  $0$  to  $2$   
 $y = 2$

$\Rightarrow$

$x$	$y = -4x + 2$	$y$	$(x, y)$
-1	$y = -4 \times (-1) + 2$	6	$(-1, 6)$
0	$y = -4 \times 0 + 2$	2	$(0, 2)$
1	$y = -4 \times 1 + 2$	-2	$(1, -2)$

**a)** Complete this table of values for the linear rule  $y = x - 1$

$x$	$y = x - 1$	$y$	$(x, y)$
-2	$y = -2 - 1$	-3	$(-2, -3)$
-1	$y = -1 - 1$	-2	$(-1, -2)$
0	$y = 0 - 1$	-1	
1			
2			

**b)** Complete this table of values for the linear rule  $y = 6x$

$x$	$y = 6x$	$y$	$(x, y)$
-2	$y = 6 \times (-2)$	-12	$(-2, -12)$
-1			
0			
1			
2			

**c)** Complete this table of values for the linear rule  $y = x + 7$

$x$	$y = x + 7$	$y$	$(x, y)$
-7	$y = -7 + 7$	0	$(-7, 0)$
-2			
0			
2			
7			

**d)** Complete this table of values for the linear rule  $y = x - 4$

$x$	$y = x - 4$	$y$	$(x, y)$
-4	$y = -4 - 4$	-8	$(-4, -8)$
-2			
0			
2			
4			

**e)** Complete this table of values for the linear rule  $y = -x - 3$

$x$	$y = -x - 3$	$y$	$(x, y)$
-3	$y = -(-3) - 3$	0	$(-3, 0)$
-1			
0			
1			
3			

**f)** Complete this table of values for the linear rule  $y = -5x + 1$

$x$	$y = -5x + 1$	$y$	$(x, y)$
-2	$y = -5 \times (-2) + 1$	11	$(-2, 11)$
-1			
0			
1			
2			

**Skill 21.2** Graphing lines of equations  $x = \text{constant}$  and  $y = \text{constant}$  on a Cartesian plane (e.g.  $x = 1$ ,  $y = 2$ ).

MM5.2 1 1 22 33 44  
MM6.1 1 1 22 33 44

- Complete the table of values for the equation. (see skill 21.1, page 223)
- Plot each point on the Cartesian plane.
- Draw the line that joins these points.
- Label the line with the rule.

*Hints: The lines of equation  $x = \text{constant}$  are vertical lines.*

*The lines of equation  $y = \text{constant}$  are horizontal lines.*

**Q.** Graph the line of equation  $y = 4$  by first completing this table of values.

[Label the line with the rule.]

$x$	-2	-1	0	1	2
$y$	4				
$(x,y)$	(-2,4)	( , )	( , )	( , )	( , )

**A.**

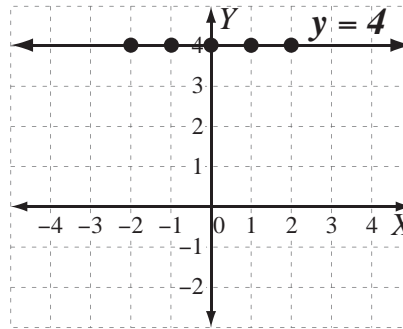
$x$	-2	-1	0	1	2
$y$	4	4	4	4	4
$(x,y)$	(-2,4)	(-1,4)	(0,4)	(1,4)	(2,4)

$y = 4$ , no matter the value of  $x$ .

Plot the points.

Join the points with a line.

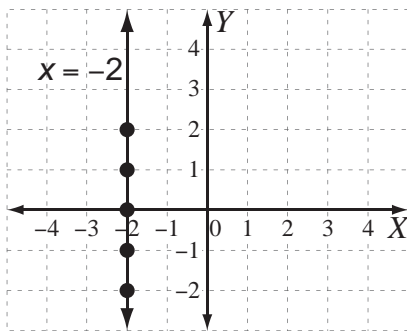
Label the line with the rule  $y = 4$



**a)** Graph the line of equation  $x = -2$  by first completing this table of values.

[Label the line with the rule.]

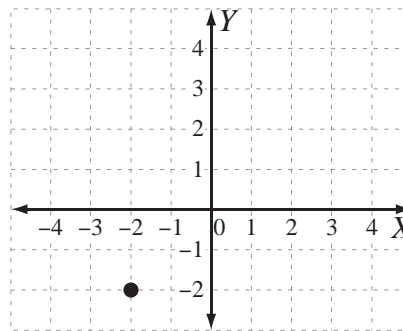
$x$	-2	-2	-2	-2	-2
$y$	-2	-1	0	1	2
$(x,y)$	(-2,-2)	(-2,-1)	(-2,0)	(-2,1)	(-2,2)



**b)** Graph the line of equation  $y = -2$  by first completing this table of values.

[Label the line with the rule.]

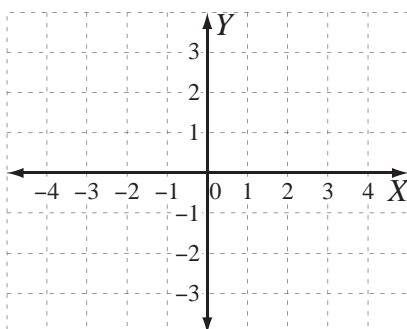
$x$	-2	-1	0	1	2
$y$	-2				
$(x,y)$	(-2,-2)	( , )	( , )	( , )	( , )



**c)** Graph the line of equation  $y = -3$  by first completing this table of values.

[Label the line with the rule.]

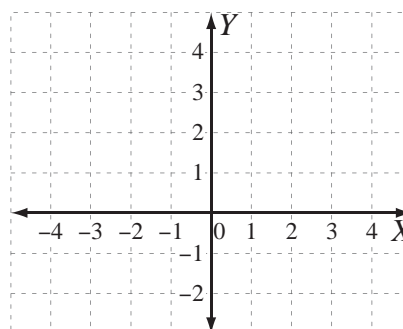
$x$	-2	-1	0	1	2
$y$	-3				



**d)** Graph the line of equation  $x = 3$  by first completing this table of values.

[Label the line with the rule.]

$x$	3				
$y$	-2	-1	0	1	2



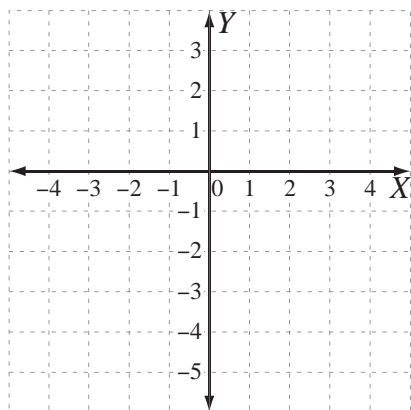
**Skill 21.3** Graphing lines of equation  $y = mx + c$  on a Cartesian plane  
(e.g.  $y = 3x + 2$ ) (1).

- Complete the table of values for the equation. (see skill 21.1, page 223)
- Plot each point on the Cartesian plane.
- Draw the line that joins these points.
- Label the line with the rule.

**Q.** Graph the line of equation  $y = -2x - 1$  by first completing this table of values.

[Label the line with the rule.]

$x$	-2	-1	0	1	2
$y$	3				
$(x,y)$	(-2,3)	( , )	( , )	( , )	( , )



**A.**  $y = -2x - 1 = -2 \times x - 1$   $2x = 2 \times x$

$x = -1 \Rightarrow y = -2 \times -1 - 1 = 1 \Rightarrow (-1, 1)$

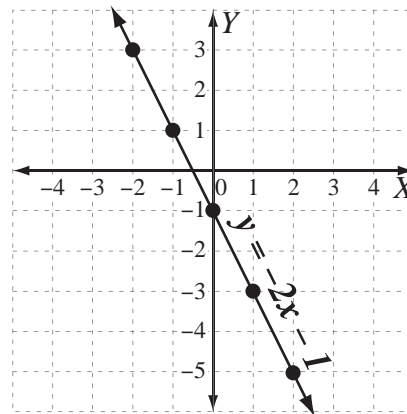
$x = 0 \Rightarrow y = -2 \times 0 - 1 = -1 \Rightarrow (0, -1)$

$x = 1 \Rightarrow y = -2 \times 1 - 1 = -3 \Rightarrow (1, -3)$

$x = 2 \Rightarrow y = -2 \times 2 - 1 = -5 \Rightarrow (2, -5)$

$x$	-2	-1	0	1	2
$y$	3	1	-1	-3	-5
$(x,y)$	(-2,3)	(-1,1)	(0,-1)	(1,-3)	(2,-5)

Complete the table of values.



Plot the points.

Join the points with a line.

Label the line with the rule  $y = -2x - 1$

**a)** Graph the line of equation  $y = -x$  by first completing this table of values.

[Label the line with the rule.]

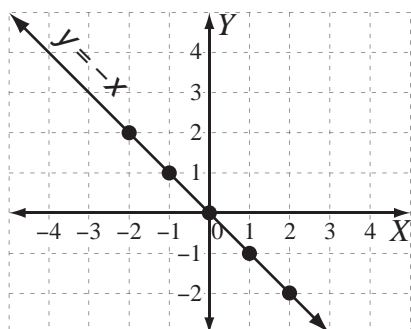
$x = -1 \Rightarrow y = -(-1) = 1 \Rightarrow (-1, 1)$

$x = 0 \Rightarrow y = -0 = 0 \Rightarrow (0, 0)$

$x = 1 \Rightarrow y = -1 \Rightarrow (1, -1)$

$x = 2 \Rightarrow y = -2 \Rightarrow (2, -2)$

$x$	-2	-1	0	1	2
$y$	2	1	0	-1	-2
$(x,y)$	(-2,2)	(-1, 1)	(0, 0)	(1, -1)	(2, -2)



**b)** Graph the line of equation  $y = x - 4$  by first completing this table of values.

[Label the line with the rule.]

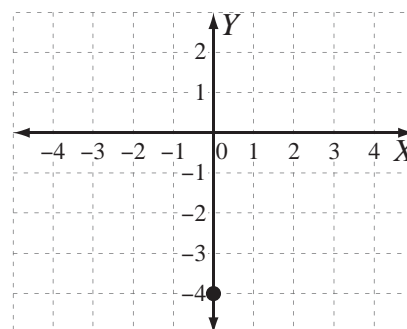
$x = 1 \Rightarrow y = 1 - 4 = -3 \Rightarrow (1, -3)$

$x = 2 \Rightarrow y = \quad \Rightarrow$

$x = 3 \Rightarrow y = \quad \Rightarrow$

$x = 4 \Rightarrow y = \quad \Rightarrow$

$x$	0	1	2	3	4
$y$	-4				
$(x,y)$	(0,-4)	( , )	( , )	( , )	( , )



**Skill 21.3** Graphing lines of equation  $y = mx + c$  on a Cartesian plane  
(e.g.  $y = 3x + 2$ ) (2).

MM5.2 1 1 22 33 44  
MM6.1 1 1 22 33 44

- c)** Graph the line of equation  $y = 2x$  by first completing this table of values.

[Label the line with the rule.]

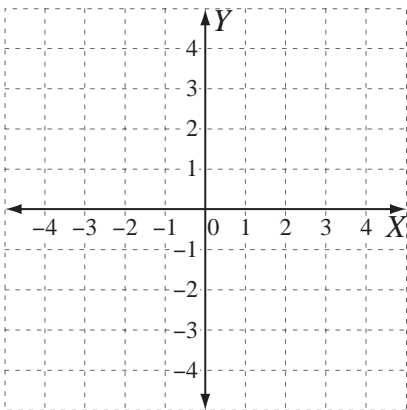
$$x = -1 \Rightarrow y = 2 \times -1 = -2 \Rightarrow (-1, -2)$$

$$x = 0 \Rightarrow y = \quad \Rightarrow$$

$$x = 1 \Rightarrow y = \quad \Rightarrow$$

$$x = 2 \Rightarrow y = \quad \Rightarrow$$

$x$	-2	-1	0	1	2
$y$	-4				



- d)** Graph the line of equation  $y = -x + 5$  by first completing this table of values.

[Label the line with the rule.]

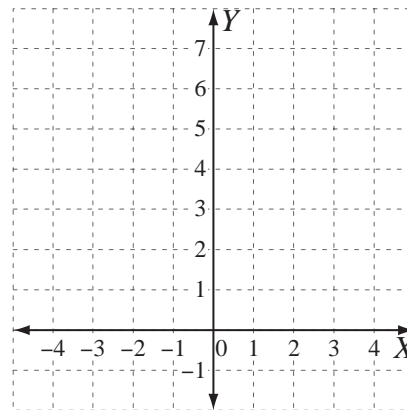
$$x = -1 \Rightarrow y = -(-1) + 5 = 6 \Rightarrow (-1, 6)$$

$$x = 0 \Rightarrow y = \quad \Rightarrow$$

$$x = 1 \Rightarrow y = \quad \Rightarrow$$

$$x = 2 \Rightarrow y = \quad \Rightarrow$$

$x$	-2	-1	0	1	2
$y$	7				



- e)** Graph the line of equation  $y = 3x - 2$  by first completing this table of values.

[Label the line with the rule.]

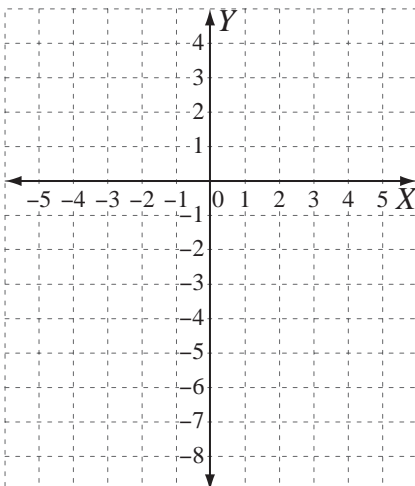
$$x = -1 \Rightarrow y = 3 \times -1 - 2 = -5 \Rightarrow (-1, -5)$$

$$x = 0 \Rightarrow y = \quad \Rightarrow$$

$$x = 1 \Rightarrow y = \quad \Rightarrow$$

$$x = 2 \Rightarrow y = \quad \Rightarrow$$

$x$	-2	-1	0	1	2
$y$	-8				



- f)** Graph the line of equation  $y = -2x - 3$  by first completing this table of values.

[Label the line with the rule.]

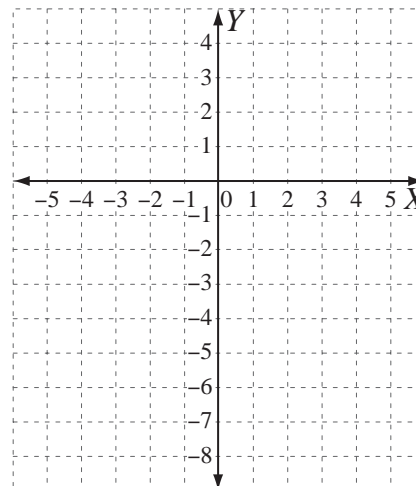
$$x = -1 \Rightarrow y = \quad \Rightarrow$$

$$x = 0 \Rightarrow y = \quad \Rightarrow$$

$$x = 1 \Rightarrow y = \quad \Rightarrow$$

$$x = 2 \Rightarrow y = \quad \Rightarrow$$

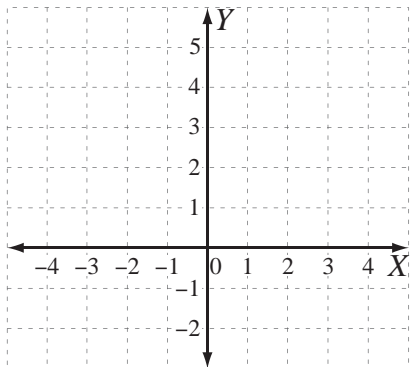
$x$	-2	-1	0	1	2
$y$	1				



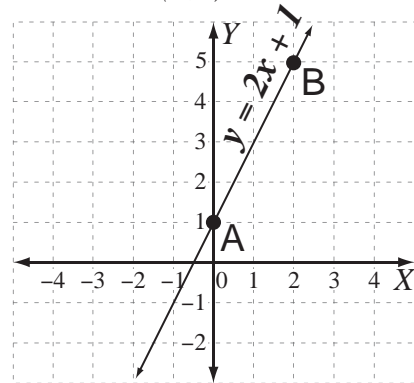
- Substitute the given value for  $x$  and solve the equation for  $y$ . (see skill 21.1, page 223)  
OR
- Substitute the given value for  $y$  into the rule and solve the equation for  $x$ .  
(see skill 20.9, page 219)
- Complete the missing coordinate.
- Plot the points on the graph.

**Q.** Complete the missing coordinates given that A and B lie on the line defined by the rule  $y = 2x + 1$ . Plot the points and draw the line.

A(0, ) , B(, 5)



**A.**  $y = 2x + 1 = 2 \times x + 1$   $2x = 2 \times x$   
 $x = 0 \Rightarrow y = 2 \times 0 + 1$  Substitute  $x = 0$  into the rule.  
 $y = 1$   
 $\Rightarrow A(0, 1)$   
 $y = 5 \Rightarrow 5 = 2 \times x + 1$  Substitute  $y = 5$  into the rule.  
 $5 - 1 = 2x + 1 - 1$   
 $2x = 4$   
 $2x \div 2 = 4 \div 2$   
 $x = 2$  Solve for  $x$ .  
 $\Rightarrow B(2, 5)$



Plot the points A and B. Draw the line by joining A and B.

**a)** Complete the missing coordinates given that M, N and P lie on the line defined by the rule  $y = -x + 3$ . Plot the points and draw the line.

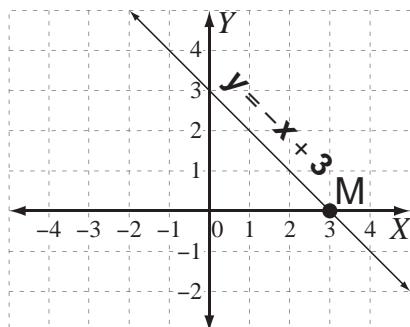
M(, 0), N(-1, ) , P(1, )

$y = 0 \Rightarrow -x + 3 = 0 \Rightarrow -x + 3 - 3 = 0 - 3$

$\Rightarrow -x = -3 \Rightarrow x = 3 \Rightarrow M(3, 0)$

$x = -1 \Rightarrow y =$

$x = 1 \Rightarrow y =$



**b)** Complete the missing coordinates given that D, E and F lie on the line defined by the rule  $y = 3x - 4$ . Plot the points and draw the line.

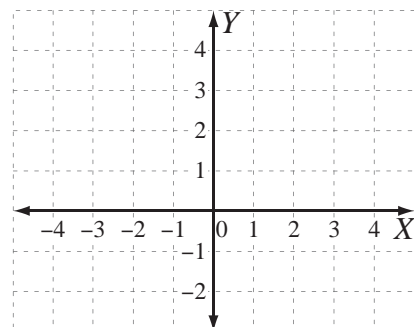
D(, 2), E(1, ) , F( $\frac{2}{3}$ , )

$y = 2 \Rightarrow$

$\Rightarrow$

$x = 1 \Rightarrow y =$

$x = \frac{2}{3} \Rightarrow y =$



- c)** Complete the missing coordinates given that B, C and D lie on the line defined by the rule  $y = 4x - 5$ . Plot the points and draw the line.

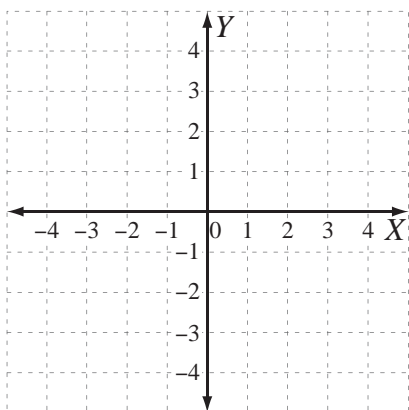
B(0, ) , C(, 3), D(, 1)

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.....

.....



- d)** Complete the missing coordinates given that S, T and U lie on the line defined by the rule  $y = x + 6$ . Plot the points and draw the line.

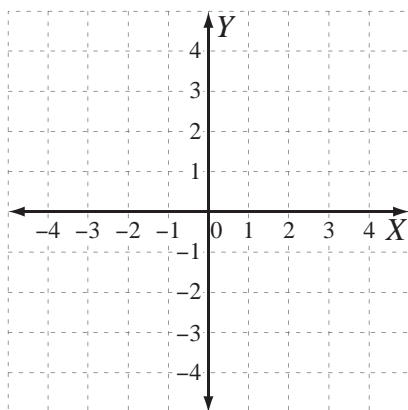
S(, 2), T(-2, ) , U(-1, )

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- e)** Complete the missing coordinates given that G, H and I lie on the line defined by the rule  $y = \frac{1}{4}x + 2$ . Plot the points and draw the line.

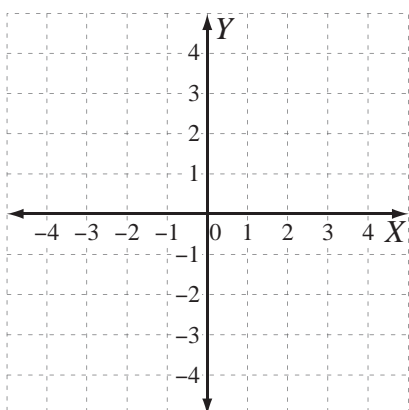
G(0, ) , H(-4, ) , I(, 3)

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- f)** Complete the missing coordinates given that A, B and C lie on the line defined by the rule  $y = 2x - \frac{1}{2}$ . Plot the points and draw the line.

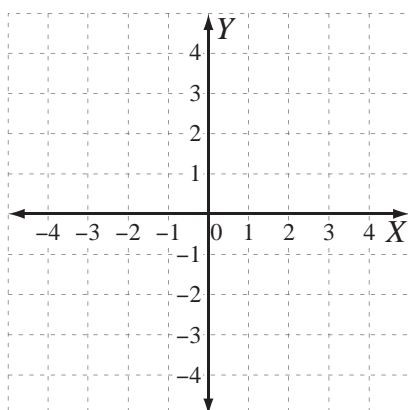
A(,  $\frac{3}{2}$ ) , B(2, ) , C(-1, )

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**Skill 21.5** Deciding if a point is on a line of a given rule.

MM5.2 1 1 2 2 3 3 4 4  
MM6.1 1 1 2 2 3 3 4 4

- Substitute the values of the coordinates  $x$  and  $y$  of the point into the given rule.
- Simplify both sides of the equation.
- Check if the statement is true, which means that the point of coordinates  $(x,y)$  lies on the line defined by the given rule.

**Q.** Which line does the point  $(1,-2)$  lie on?

- A)  $y = x$   
B)  $y = 2x - 3$   
C)  $y = 6x - 8$

**A.** A)  $y = x \Rightarrow -2 = 1$  (*false*)  
B)  $y = 2x - 3 \Rightarrow -2 = 2 \times 1 - 3 \Rightarrow -2 = -1$  (*false*)  
C)  $y = 6x - 8 \Rightarrow -2 = 6 \times 1 - 8 \Rightarrow -2 = -2$  (*true*)

Substitute  $x = 1$  and  $y = -2$  into each rule.

The answer is **C**.

**a)** Which of these points lies on the line defined by the rule  $y = 4x + 3$ ?

- A  $(-3,0)$   
B  $(2,2)$   
C  $(-1,-1)$

A)  $x = -3, y = 0 \Rightarrow 0 = 4 \times -3 + 3$

$\Rightarrow 0 = -9$  (*false*)

B)  $x = 2, y = 2 \Rightarrow 2 = 4 \times 2 + 3$

$\Rightarrow 2 = 11$  (*false*)

C)  $x = -1, y = -1 \Rightarrow -1 = 4 \times -1 + 3$

$\Rightarrow -1 = -1$  (*true*)

**C**

**b)** Which of these points lies on the line defined by the rule  $y = -2x + 5$ ?

- A  $(3,-3)$   
B  $(-1,7)$   
C  $(0,-2)$

A)  $x = 3, y = -3 \Rightarrow$

$\Rightarrow$

B)  $x = -1, y = 7 \Rightarrow$

$\Rightarrow$

C)  $x = 0, y = -2 \Rightarrow$

$\Rightarrow$

**c)** Which line does the point  $(2,-1)$  lie on?

- A)  $y = x + 1$   
B)  $y = 5 - 3x$   
C)  $y = 2x$

A)  $y = x + 1 \Rightarrow$

$\Rightarrow$

B)  $y = 5 - 3x \Rightarrow$

$\Rightarrow$

C)  $y = 2x \Rightarrow$

$\Rightarrow$

**d)** Which line does the point  $(-1,1)$  lie on?

- A)  $y = -3x$   
B)  $y = 7 - 4x$   
C)  $y = 5x + 6$

A)  $y = -3x \Rightarrow$

$\Rightarrow$

B)  $y = 7 - 4x \Rightarrow$

$\Rightarrow$

C)  $y = 5x + 6 \Rightarrow$

$\Rightarrow$

## Skill 21.6 Finding the $x$ -intercept and the $y$ -intercept of a linear graph (1).

MM5.2 1 1 2 2 3 3 4 4  
MM6.1 1 1 2 2 3 3 4 4

- To find the  $x$ -intercept, substitute  $y = 0$  into the rule.
- Solve for  $x$ .
- To find the  $y$ -intercept, substitute  $x = 0$  into the rule.
- Solve for  $y$ .

**Q.** Find the  $x$ -intercept and the  $y$ -intercept of the line defined by the equation  $2x - 5y = 10$ .

**A.**  $x$ -intercept  $\Rightarrow y = 0$   
 $\Rightarrow 2x - 0 = 10$   
 $2x \div 2 = 10 \div 2$   
 $x = 5$   
 $x$ -intercept is **(5,0)**  
 $y$ -intercept  $\Rightarrow x = 0$   
 $\Rightarrow 0 - 5y = 10$   
 $-5y \div -5 = 10 \div -5$   
 $y = -2$   
 $y$ -intercept is **(0,-2)**

**a)** Find the  $x$ -intercept of the line defined by the equation  $y = -3x + 6$

$$y = 0 \Rightarrow 0 = -3x + 6$$

$$0 - 6 = -3x + 6 - 6$$

$$-3x = -6$$

$$-3x \div -3 = -6 \div -3$$

$$x = 2 \Rightarrow x\text{-intercept is } \boxed{(2,0)}$$

**b)** Find the  $x$ -intercept of the line defined by the equation  $y = 2x + 8$

$$y = 0 \Rightarrow$$

$$\Rightarrow x\text{-intercept is } \boxed{\phantom{00}}$$

**c)** Find the  $y$ -intercept of the line defined by the equation  $y = 7x - 3$

$$x = 0 \Rightarrow y = 0 - 3$$

$$\Rightarrow y = -3 \Rightarrow y\text{-intercept is } \boxed{\phantom{00}}$$

**d)** Find the  $y$ -intercept of the line defined by the equation  $y = -5x + 4$

$$x = 0 \Rightarrow$$

$$\Rightarrow \Rightarrow y\text{-intercept is } \boxed{\phantom{00}}$$

**e)** Find the  $x$ -intercept of the line defined by the equation  $3x - 2y = -12$

$$\Rightarrow x\text{-intercept is } \boxed{\phantom{00}}$$

**f)** Find the  $y$ -intercept of the line defined by the equation  $4y - x = 16$

$$\Rightarrow y\text{-intercept is } \boxed{\phantom{00}}$$



**Skill 21.6** Finding the  $x$ -intercept and the  $y$ -intercept of a linear graph (2).

MM5.2 1 1 2 3 3 4 4  
MM6.1 1 1 2 2 3 3 4 4

**g)** Find the  $x$ -intercept of the line defined by the equation  $y = -2x + 10$

.....  
.....  
.....  
 $\Rightarrow x$ -intercept is

**h)** Find the  $y$ -intercept of the line defined by the equation  $y = 3x - 9$

.....  
.....  
.....  
 $\Rightarrow y$ -intercept is

**i)** Find the  $y$ -intercept of the line defined by the equation  $y = 6x - 8$

.....  
.....  
.....  
 $\Rightarrow y$ -intercept is

**j)** Find the  $x$ -intercept of the line defined by the equation  $y = -5x + 1$

.....  
.....  
.....  
 $\Rightarrow x$ -intercept is

**k)** Find the  $y$ -intercept of the line defined by the equation  $y = -4x + 5$

.....  
.....  
.....  
 $\Rightarrow y$ -intercept is

**l)** Find the  $y$ -intercept of the line defined by the equation  $y = 3x - 2$

.....  
.....  
.....  
 $\Rightarrow y$ -intercept is

**m)** Find the  $x$ -intercept of the line defined by the equation  $4x - y = 8$

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.....  
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 $\Rightarrow x$ -intercept is

**n)** Find the  $x$ -intercept of the line defined by the equation  $2y - x = 7$

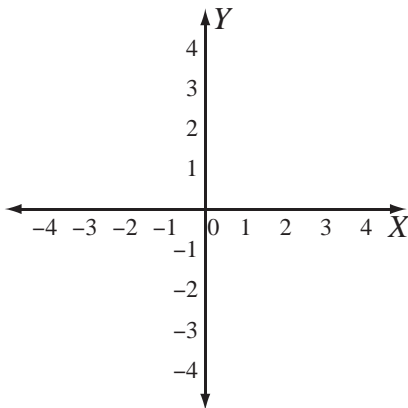
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 $\Rightarrow x$ -intercept is

## Skill 21.7 Sketching a linear graph by finding the x-intercept and the y-intercept (1).

MM5.2 11 22 33 44  
MM6.1 11 22 33 44

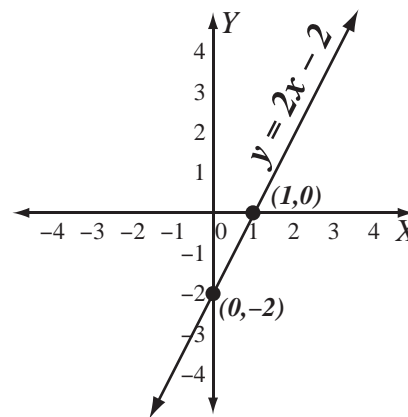
- Find the x-intercept and the y-intercept. (see skill 21.6, page 230)
- Mark each intercept point on the Cartesian plane.
- Draw the line that joins these points.
- Label the line with the rule.

**Q.** Sketch the line of equation  $y = 2x - 2$  by marking the x-intercept and the y-intercept.  
[Label the graph with the rule.]



**A.**  $x\text{-intercept} \Rightarrow y = 0 \Rightarrow 2x - 2 = 0$   
 $2x - 2 + 2 = 0 + 2$   
 $2x + 2 = 2 \div 2$   
 $x = 1$   
 $\Rightarrow x\text{-intercept is } (1, 0)$

$y\text{-intercept} \Rightarrow x = 0 \Rightarrow y = 2 \times 0 - 2$   
 $\Rightarrow y = -2$   
 $\Rightarrow y\text{-intercept is } (0, -2)$



Mark the intercept points.

Join the points with a line.

Label the line with the rule  $y = 2x - 2$

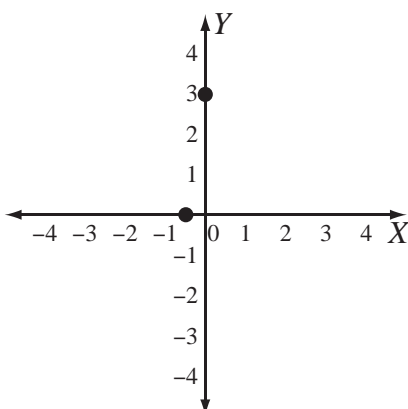
**a)** Sketch the line of equation  $y = 6x + 3$  by marking the x-intercept and the y-intercept.  
[Label the graph with the rule.]

$$y = 0 \Rightarrow 6x + 3 = 0 \Rightarrow 6x = -3 \Rightarrow x = -\frac{1}{2}$$

$$\Rightarrow x\text{-intercept is } \left(-\frac{1}{2}, 0\right)$$

$$x = 0 \Rightarrow y = 6 \times 0 + 3 \Rightarrow y = 3$$

$$\Rightarrow y\text{-intercept is } (0, 3)$$



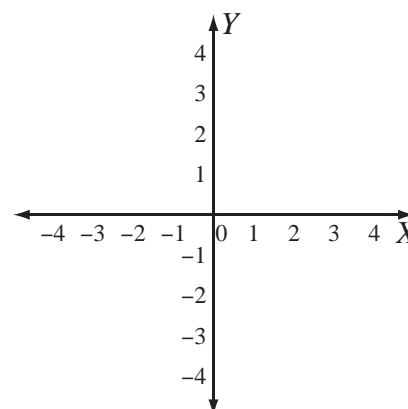
**b)** Sketch the line of equation  $y = -3x + 4$  by marking the x-intercept and the y-intercept.  
[Label the graph with the rule.]

$$y = 0 \Rightarrow$$

$$\Rightarrow x\text{-intercept is}$$

$$x = 0 \Rightarrow$$

$$\Rightarrow y\text{-intercept is}$$



**Skill 21.7** Sketching a linear graph by finding the  $x$ -intercept and the  $y$ -intercept (2).

MM5.2 1 1 2 3 3 4 4  
MM6.1 1 1 2 2 3 3 4 4

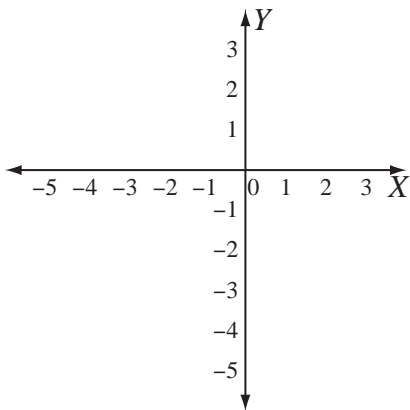
- c)** Sketch the line of equation  $y = -x - 5$  by marking the  $x$ -intercept and the  $y$ -intercept.  
[Label the graph with the rule.]

$y = 0 \Rightarrow$   
.....

$\Rightarrow x$ -intercept is  
.....

$x = 0 \Rightarrow$   
.....

$\Rightarrow y$ -intercept is  
.....



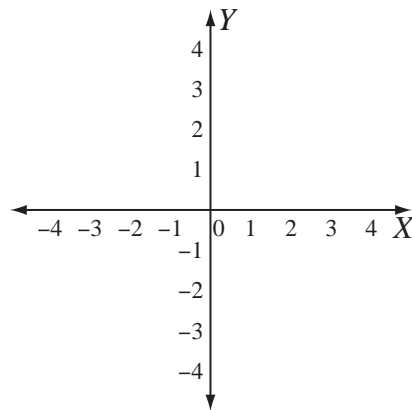
- d)** Sketch the line of equation  $y = 9x - 3$  by marking the  $x$ -intercept and the  $y$ -intercept.  
[Label the graph with the rule.]

$y = 0 \Rightarrow$   
.....

$\Rightarrow x$ -intercept is  
.....

$x = 0 \Rightarrow$   
.....

$\Rightarrow y$ -intercept is  
.....



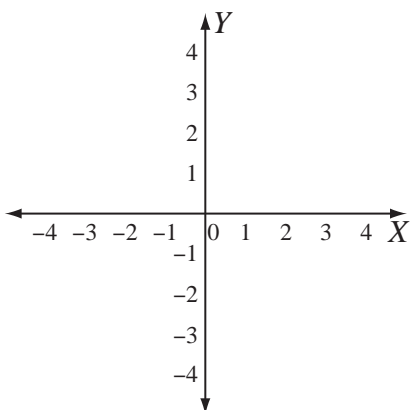
- e)** Sketch the line of equation  $y = -4x - 2$  by marking the  $x$ -intercept and the  $y$ -intercept.  
[Label the graph with the rule.]

$y = 0 \Rightarrow$   
.....

$\Rightarrow x$ -intercept is  
.....

$x = 0 \Rightarrow$   
.....

$\Rightarrow y$ -intercept is  
.....



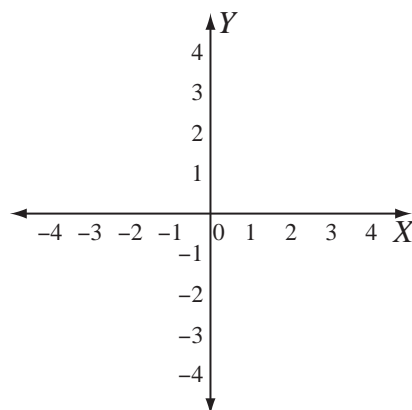
- f)** Sketch the line of equation  $y = -2x + 4$  by marking the  $x$ -intercept and the  $y$ -intercept.  
[Label the graph with the rule.]

$y = 0 \Rightarrow$   
.....

$\Rightarrow x$ -intercept is  
.....

$x = 0 \Rightarrow$   
.....

$\Rightarrow y$ -intercept is  
.....



## Skill 21.8 Finding the gradient of a line by using the rise/run formula (1).

MM5.2 11 22 33 44  
MM6.1 11 22 33 44

- Choose two convenient points on the graph and draw a right-angled triangle using the line of the graph as the hypotenuse.
- Measure the vertical rise of the graph (the vertical side of the triangle):
  - positive value if the graph rises from left to right
  - negative value if the graph drops from left to right
- Measure the horizontal run of the graph (the horizontal side of the triangle):
  - always a positive value.

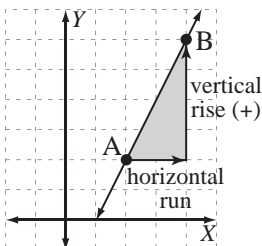
*Hints: The gradient gives an indication of how steep a line is.*

*The gradient is positive if the graph rises from left to right.*

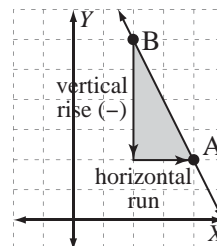
*The gradient is negative if the graph falls from left to right.*

$$\text{Gradient} = \frac{\text{vertical rise}}{\text{horizontal run}}$$

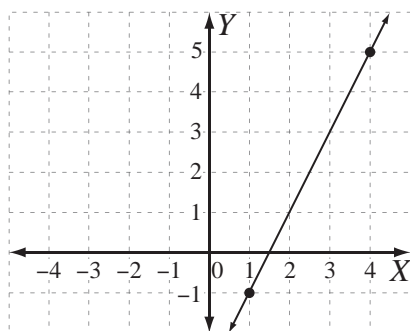
Positive gradient



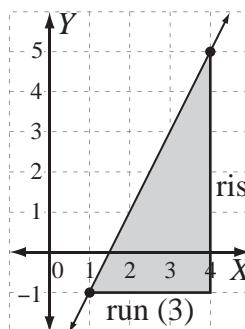
Negative gradient



- Q.** Find the gradient of the line passing through the points (1, -1) and (4, 5).



**A.**



Draw a right-angled triangle.

Measure the rise and run.

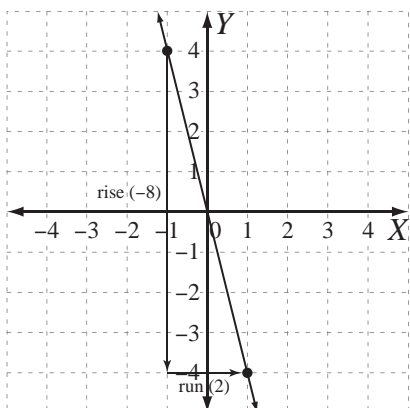
$$\begin{aligned} \text{rise} &= 6 \\ \text{run} &= 3 \\ \text{gradient} &= \frac{\text{rise}}{\text{run}} = \frac{6}{3} = 2 \end{aligned}$$

- a)** Find the gradient of the line passing through the points (-1, 4) and (1, -4).

$$\text{rise} = -8$$

$$\text{run} = 2$$

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = -\frac{8}{2} = \boxed{\phantom{00}}$$

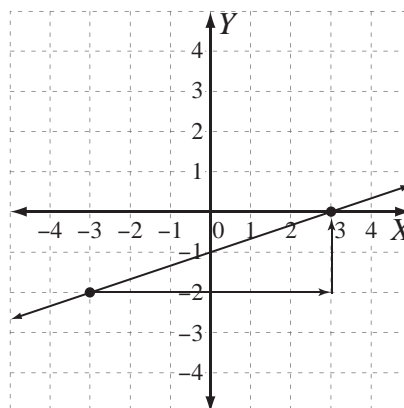


- b)** Find the gradient of the line passing through the points (-3, -2) and (3, 0).

$$\text{rise} = 2$$

$$\text{run} = 6$$

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \phantom{00} = \boxed{\phantom{00}}$$



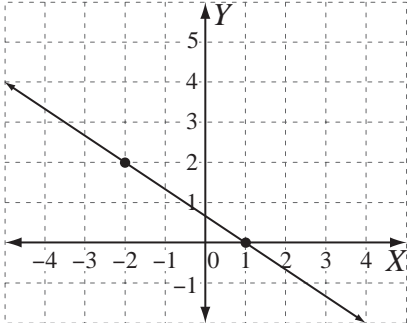
**Skill 21.8** Finding the gradient of a line by using the rise/run formula (2).

MM5.2 11 22 33 44  
MM6.1 11 22 33 44

- c)** Find the gradient of the line passing through the points  $(-2,2)$  and  $(1,0)$ .

*rise* = \_\_\_\_\_ *run* = \_\_\_\_\_

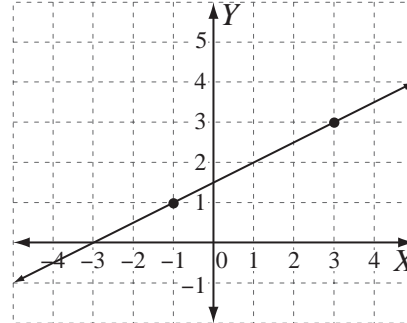
*gradient* = \_\_\_\_\_ = \_\_\_\_\_ =



- d)** Find the gradient of the line passing through the points  $(-1,1)$  and  $(3,3)$ .

*rise* = \_\_\_\_\_ *run* = \_\_\_\_\_

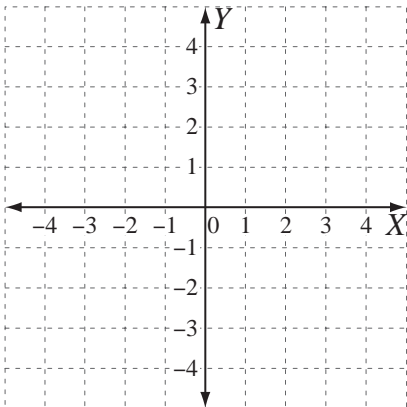
*gradient* = \_\_\_\_\_ = \_\_\_\_\_ =



- e)** Graph the line passing through the points  $(-1,1)$  and  $(3,2)$ . What is the gradient of the line?

*rise* = \_\_\_\_\_ *run* = \_\_\_\_\_

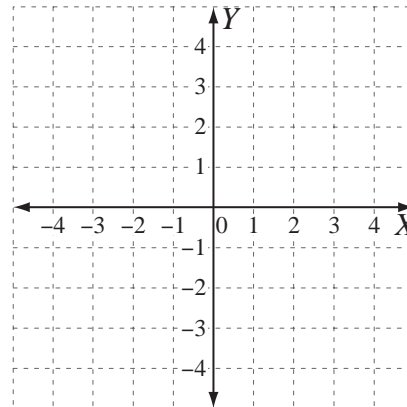
*gradient* = \_\_\_\_\_ = \_\_\_\_\_ =



- f)** Graph the line passing through the points  $(-2,4)$  and  $(2,-2)$ . What is the gradient of the line?

*rise* = \_\_\_\_\_ *run* = \_\_\_\_\_

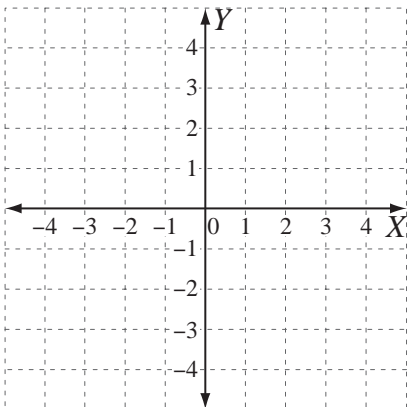
*gradient* = \_\_\_\_\_ = \_\_\_\_\_ =



- g)** Graph the line passing through the points  $(-1,-2)$  and  $(4,4)$ . What is the gradient of the line?

*rise* = \_\_\_\_\_ *run* = \_\_\_\_\_

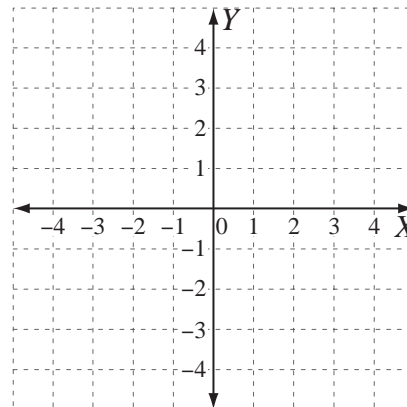
\_\_\_\_\_ =



- h)** Graph the line passing through the points  $(-4,3)$  and  $(2,-4)$ . What is the gradient of the line?

*rise* = \_\_\_\_\_ *run* = \_\_\_\_\_

\_\_\_\_\_ =



## Skill 21.9 Finding the coordinates of the midpoint of an interval.

MM5.2 11 22 33 44  
MM6.1 11 22 33 44

- Identify  $(x_1, y_1)$  and  $(x_2, y_2)$  as the coordinates of the given points.
- Write the formula for the midpoint of an interval.

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- Substitute the values of  $x_1$ ,  $x_2$ ,  $y_1$  and  $y_2$  into the formula.

**Q.** Use  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$  to find the coordinates of the midpoint  $M$  of the interval joining the points  $(6, -4)$  and  $(2, 8)$ .

**A.**  $(x_1, y_1) = (6, -4) \Rightarrow x_1 = 6$  and  $y_1 = -4$

$(x_2, y_2) = (2, 8) \Rightarrow x_2 = 2$  and  $y_2 = 8$

$$\frac{x_1 + x_2}{2} = \frac{6 + 2}{2} = \frac{8}{2} = 4$$

$$\frac{y_1 + y_2}{2} = \frac{-4 + 8}{2} = \frac{4}{2} = 2$$

The coordinates of the midpoint  $M$  are **(4, 2)**

**a)** Use  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$  to find the coordinates of the midpoint  $M$  of the interval joining the points  $(-5, -4)$  and  $(1, -3)$ .

$$x_1 = \quad, y_1 = \quad, x_2 = \quad, y_2 = \quad$$

$$\frac{x_1 + x_2}{2} = \quad = \quad$$

$$\frac{y_1 + y_2}{2} = \quad = \quad \Rightarrow \boxed{\quad}$$

**b)** Use  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$  to find the coordinates of the midpoint  $M$  of the interval joining the points  $(3, 7)$  and  $(-2, 3)$ .

$$x_1 = \quad, y_1 = \quad, x_2 = \quad, y_2 = \quad$$

$$\frac{x_1 + x_2}{2} = \quad = \quad$$

$$\frac{y_1 + y_2}{2} = \quad = \quad \Rightarrow \boxed{\quad}$$

**c)** Use  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$  to find the coordinates of the midpoint  $M$  of the interval joining the points  $(5, 0)$  and  $(7, 9)$ .

$$x_1 = \quad, y_1 = \quad, x_2 = \quad, y_2 = \quad$$

$$= \quad = \quad$$

$$= \quad = \quad \Rightarrow \boxed{\quad}$$

**d)** Use  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$  to find the coordinates of the midpoint  $M$  of the interval joining the points  $(1, -3)$  and  $(-4, 6)$ .

$$x_1 = \quad, y_1 = \quad, x_2 = \quad, y_2 = \quad$$

$$= \quad = \quad$$

$$= \quad = \quad \Rightarrow \boxed{\quad}$$

**e)** Use  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$  to find the coordinates of the midpoint  $M$  of the interval joining the points  $(-2, -5)$  and  $(-1, 0)$ .

$$x_1 = \quad, y_1 = \quad, x_2 = \quad, y_2 = \quad$$

$$= \quad = \quad$$

$$= \quad = \quad \Rightarrow \boxed{\quad}$$

**f)** Use  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$  to find the coordinates of the midpoint  $M$  of the interval joining the points  $(0, 6)$  and  $(-2, -4)$ .

$$x_1 = \quad, y_1 = \quad, x_2 = \quad, y_2 = \quad$$

$$= \quad = \quad$$

$$= \quad = \quad \Rightarrow \boxed{\quad}$$

**Skill 21.10** Rewriting a linear equation in the gradient-intercept form.

MM5.2 1 1 2 2 3 3 4 4  
MM6.1 1 1 2 2 3 3 4 4

- Use the inverse operations of addition, subtraction, multiplication and/or division to rearrange the terms in the rule:
    - y on the left hand side of the equal sign without coefficient
    - x on the right hand side of the equal sign.
- Hint: The general form of a linear function (rule) is  $y = mx + c$   
The coefficient of x is the gradient of the graph (m).  
The number that is not attached to either x or y is the y-intercept (c).*

**Q.** Write the equation  $-x + 2y = -3$  in the gradient-intercept form  $y = mx + c$ , where  $m$  represents the gradient and  $c$  the y-intercept.

**A.**

$$-x + 2y = -3$$

$$-x + x + 2y = -3 + x$$

$$2y = x - 3$$

$$\frac{2y}{2} = \frac{x - 3}{2}$$

$$y = \frac{1}{2}x - \frac{3}{2}$$

**a)** Write the equation  $5 - y = 4x$  in the gradient-intercept form  $y = mx + c$ , where  $m$  represents the gradient and  $c$  the y-intercept.

$$5 - y - 5 = 4x - 5 \Rightarrow -y = 4x - 5$$

$$- -y = -(4x - 5)$$

$$y = -4x - (-5) \quad \boxed{y = -4x + 5}$$

**b)** Write the equation  $x - 5y = 5$  in the gradient-intercept form  $y = mx + c$ , where  $m$  represents the gradient and  $c$  the y-intercept.

$$y = \boxed{\phantom{y =}}$$

**c)** Write the equation  $3x + 2y = 1$  in the gradient-intercept form  $y = mx + c$ , where  $m$  represents the gradient and  $c$  the y-intercept.

$$y = \boxed{\phantom{y =}}$$

**d)** Write the equation  $4x - 2y = 3$  in the gradient-intercept form  $y = mx + c$ , where  $m$  represents the gradient and  $c$  the y-intercept.

$$y = \boxed{\phantom{y =}}$$

**e)** Write the equation  $-2x - y = 6$  in the gradient-intercept form  $y = mx + c$ , where  $m$  represents the gradient and  $c$  the y-intercept.

$$y = \boxed{\phantom{y =}}$$

**f)** Write the equation  $3x + 4y = 12$  in the gradient-intercept form  $y = mx + c$ , where  $m$  represents the gradient and  $c$  the y-intercept.

$$y = \boxed{\phantom{y =}}$$

**Skill 21.11** Finding the gradient, the x-intercept and the y-intercept of an equation written in the gradient-intercept form  $y = mx + c$  (1).

MM5.2 11 22 3 44  
MM6.1 11 22 33 44

- Write the equation given in the table in the gradient-intercept form  $y = mx + c$  (see skill 21.10, page 237)
- Identify the gradient ( $m$ ) of the linear function as the coefficient of  $x$ .
- Identify the y-intercept of the linear function as the constant ( $c$ ).
- Find the x-intercept of the function. (see skill 21.6, page 230)
- Fill in the table.

**Q.** Complete the following table:

equation	gradient ( $m$ )	x-intercept	y-intercept ( $c$ )
$y = -2x - 10$			

**A.**  $y = -2x - 10$

$$y = -2x - 10$$

$$y = mx + c \Rightarrow m = -2 \text{ (gradient)}$$

$$\Rightarrow c = -10 \text{ (y-intercept)}$$

y-intercept is  $(0, -10)$

$$x\text{-intercept} \Rightarrow y = 0$$

$$\Rightarrow -2x - 10 = 0$$

$$-2x - 10 + 10 = 0 + 10$$

$$-2x = 10$$

$$\frac{-2x}{-2} = \frac{10}{-2}$$

$$x = -5$$

x-intercept is  $(-5, 0)$

equation	gradient ( $m$ )	x-intercept	y-intercept ( $c$ )
$y = -2x - 10$	$-2$	$(-5, 0)$	$(0, -10)$

**a)** Complete the following table:

equation	gradient ( $m$ )	x-intercept	y-intercept ( $c$ )
$y = 2x - 6$	$2$	$(3, 0)$	$(0, -6)$

$$y = 2x - 6$$

$$y = mx + c \Rightarrow m = 2$$

$$\Rightarrow c = -6 \Rightarrow y\text{-intercept is } (0, -6)$$

$$y = 0 \Rightarrow 2x - 6 = 0$$

$$2x - 6 + 6 = 0 + 6$$

$$2x = 6$$

$$2x \div 2 = 6 \div 2$$

$$x = 3 \Rightarrow x\text{-intercept is } (3, 0)$$

**b)** Complete the following table:

equation	gradient ( $m$ )	x-intercept	y-intercept ( $c$ )
$y = -x + 5$			

$$y = -x + 5$$

$$y = mx + c \Rightarrow m =$$

$$\Rightarrow c = \Rightarrow y\text{-intercept is}$$

$$y = 0 \Rightarrow$$

$$\Rightarrow x\text{-intercept is}$$



**Skill 21.11** Finding the gradient, the  $x$ -intercept and the  $y$ -intercept of an equation written in the gradient-intercept form  $y = mx + c$  (2).

**c)** Complete the following table:

equation	gradient ( $m$ )	$x$ -intercept	$y$ -intercept ( $c$ )
$y = \frac{1}{3}x - 2$			

$y = \frac{1}{3}x - 2$

$y = mx + c \Rightarrow m =$

$\Rightarrow c = \Rightarrow y\text{-intercept is}$

$y = 0 \Rightarrow$

$\Rightarrow x\text{-intercept is}$

**d)** Complete the following table:

equation	gradient ( $m$ )	$x$ -intercept	$y$ -intercept ( $c$ )
$y = \frac{2}{5}x + 4$			

$y = \frac{2}{5}x + 4$

$y = mx + c \Rightarrow m =$

$\Rightarrow c = \Rightarrow y\text{-intercept is}$

$y = 0 \Rightarrow$

$\Rightarrow x\text{-intercept is}$

**e)** Complete the following table:

equation	gradient ( $m$ )	$x$ -intercept	$y$ -intercept ( $c$ )
$y = 5x + 3$			

$y = 5x + 3$

$y = mx + c \Rightarrow m =$

$\Rightarrow c = \Rightarrow y\text{-intercept is}$

$y = 0 \Rightarrow$

$\Rightarrow x\text{-intercept is}$

**f)** Complete the following table:

equation	gradient ( $m$ )	$x$ -intercept	$y$ -intercept ( $c$ )
$y = -2x + 1$			

$y = -2x + 1$

$y = mx + c \Rightarrow m =$

$\Rightarrow c = \Rightarrow y\text{-intercept is}$

$y = 0 \Rightarrow$

$\Rightarrow x\text{-intercept is}$

**Skill 21.12** Finding the gradient of a line when two points are given.

- Identify  $(x_1, y_1)$  and  $(x_2, y_2)$  as the coordinates of the given points.
- Write the formula for the gradient of a linear graph.
- Substitute the values of  $x_1$ ,  $x_2$ ,  $y_1$  and  $y_2$  into the formula.
- Simplify and evaluate the value of  $m$ .

$$\text{Gradient } m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

**Q.** Use  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to find the gradient of the line passing through the points  $(6, -2)$  and  $(-2, 4)$ .

**A.**  $(x_1, y_1) = (6, -2) \Rightarrow x_1 = 6$  and  $y_1 = -2$

$(x_2, y_2) = (-2, 4) \Rightarrow x_2 = -2$  and  $y_2 = 4$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{-2 - 6}$$

$$= \frac{4 - (-2)}{-2 - 6} = \frac{6}{-8} = -\frac{3}{4}$$

*Simplify:  $\div 2$*

**a)** Use  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to find the gradient of the line passing through the points  $(-1, 3)$  and  $(2, 0)$ .

$$x_1 = -1, y_1 = 3, x_2 = 2, y_2 = 0$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{2 - (-1)} = \frac{-3}{3} = \boxed{\phantom{00}}$$

**b)** Use  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to find the gradient of the line passing through the points  $(2, 1)$  and  $(-2, -7)$ .

$$x_1 = \phantom{00}, y_1 = \phantom{00}, x_2 = \phantom{00}, y_2 = \phantom{00}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \phantom{00} = \phantom{00} = \boxed{\phantom{00}}$$

**c)** Use  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to find the gradient of the line passing through the points  $(1, -1)$  and  $(3, 3)$ .

$$= \boxed{\phantom{00}}$$

**d)** Use  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to find the gradient of the line passing through the points  $(3, -1)$  and  $(-1, -2)$ .

$$= \boxed{\phantom{00}}$$

**e)** Use  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to find the gradient of the line passing through the points  $(0, 3)$  and  $(-6, 0)$ .

$$= \boxed{\phantom{00}}$$

**f)** Use  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to find the gradient of the line passing through the points  $(-4, 1)$  and  $(2, -1)$ .

$$= \boxed{\phantom{00}}$$

**g)** Use  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to find the gradient of the line passing through the points  $(3, -1)$  and  $(1, 2)$ .

$$= \boxed{\phantom{00}}$$

**h)** Use  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to find the gradient of the line passing through the points  $(-4, 0)$  and  $(-1, 5)$ .

$$= \boxed{\phantom{00}}$$

**Skill 21.13** Writing the equation of a line when two points are given (1).

MM5.2 1 1 2 2 3 3 4 4  
MM6.1 1 1 2 2 3 3 4 4

- Identify  $(x_1, y_1)$  and  $(x_2, y_2)$  as the coordinates of the given points.
- Find the gradient of the line joining the two points. (see skill 21.12, page 240)
- Write the equation  $y - y_1 = m(x - x_1)$  of a straight line.
- Substitute the values of  $y_1$ ,  $m$  and  $x_1$  into the equation.
- Simplify to rearrange the equation:
  - $y$  on its own on the left hand side of the equal sign without coefficient
  - $x$  and the remaining number on the right hand side of the equal sign

$$y - y_1 = m(x - x_1)$$

**Q.** Use  $y - y_1 = m(x - x_1)$  where  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to write the equation of the line passing through the points  $(-3, 2)$  and  $(4, -1)$ .

**A.**  $(x_1, y_1) = (-3, 2) \Rightarrow x_1 = -3$  and  $y_1 = 2$   
 $(x_2, y_2) = (4, -1) \Rightarrow x_2 = 4$  and  $y_2 = -1$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{4 - (-3)} = \frac{-3}{7}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{7} \times (x - (-3))$$

$$y - 2 = -\frac{3}{7} \times (x + 3)$$

$$y - 2 = -\frac{3x}{7} - \frac{9}{7}$$

$$y - 2 + 2 = -\frac{3x}{7} - \frac{9}{7} + 2$$

$$y = -\frac{3}{7}x + \frac{5}{7}$$

---+

+---

$-\frac{9}{7} + 2 = -\frac{9}{7} + \frac{14}{7} = \frac{5}{7}$

**a)** Use  $y - y_1 = m(x - x_1)$  where  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to write the equation of the line passing through the points  $(7, 0)$  and  $(-1, 8)$ .

$$x_1 = 7, y_1 = 0, x_2 = -1, y_2 = 8$$

$$m = \frac{8 - 0}{-1 - 7} = \frac{8}{-8} = -1$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -1 \times (x - 7)$$

$$y = -1 \times x - (-1) \times 7$$

$$y = -x + 7$$

---+

**b)** Use  $y - y_1 = m(x - x_1)$  where  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to write the equation of the line passing through the points  $(-4, 5)$  and  $(-3, 7)$ .

$$x_1 = \quad, y_1 = \quad, x_2 = \quad, y_2 = \quad$$

$$m = \quad = \quad = \quad$$

$$y - y_1 = m(x - x_1)$$

$$y =$$

**Skill 21.13** Writing the equation of a line when two points are given (2).

- c)** Use  $y - y_1 = m(x - x_1)$  where  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to write the equation of the line passing through the points (1,5) and (3,11).

$x_1 = \quad , y_1 = \quad , x_2 = \quad , y_2 = \quad$

$m = \quad = \quad = \quad$

$y - y_1 = m(x - x_1)$

$y =$

- d)** Use  $y - y_1 = m(x - x_1)$  where  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to write the equation of the line passing through the points (1,-3) and (-4,-2).

$x_1 = \quad , y_1 = \quad , x_2 = \quad , y_2 = \quad$

$m = \quad = \quad = \quad$

$y - y_1 = m(x - x_1)$

$y =$

- e)** Use  $y - y_1 = m(x - x_1)$  where  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to write the equation of the line passing through the points (4,-3) and (-4,5).

$x_1 = \quad , y_1 = \quad , x_2 = \quad , y_2 = \quad$

$m = \quad = \quad = \quad$

$y - y_1 = m(x - x_1)$

$y =$

- f)** Use  $y - y_1 = m(x - x_1)$  where  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to write the equation of the line passing through the points (-6,2) and (-2,-2).

$x_1 = \quad , y_1 = \quad , x_2 = \quad , y_2 = \quad$

$m = \quad = \quad = \quad$

$y - y_1 = m(x - x_1)$

$y =$

- g)** Use  $y - y_1 = m(x - x_1)$  where  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to write the equation of the line passing through the points (3,0) and (-1,5).

$x_1 = \quad$

$y - y_1 = m(x - x_1)$

$y - y_1 = m(x - x_1)$

$y - y_1 = m(x - x_1)$

$y =$

- h)** Use  $y - y_1 = m(x - x_1)$  where  $m = \frac{y_2 - y_1}{x_2 - x_1}$  to write the equation of the line passing through the points (2,-4) and (0,-1).

$x_1 = \quad$

$y - y_1 = m(x - x_1)$

$y - y_1 = m(x - x_1)$

$y - y_1 = m(x - x_1)$

$y =$

### Skill 21.14 Completing a table of values for a non-linear rule.

MM5.2 1 1 2 2 3 3 4 4  
MM6.1 1 1 2 2 3 3 4 4

- Substitute the variable  $x$  with the given values.
- Solve the equation for  $y$ .
- Use the order of operations rules. (see skill 16.1, page 131)
- Use the sign rules. (see skill 9.1, page 93)
- Complete the table of values for the non-linear function.

**Q.** Complete the table of values for the non-linear rule  $y = x^2 - 3$  (parabola).

$x$	-2	-1	0	1	2
$y$	1				

**A.**  $y = x^2 - 3 = x \times x - 3$   $x^2 = x \times x$

$x = -1 \Rightarrow y = -1 \times -1 - 3 = 1 - 3 \Rightarrow y = -2$

$x = 0 \Rightarrow y = 0 \times 0 - 3 = 0 - 3 \Rightarrow y = -3$

$x = 1 \Rightarrow y = 1 \times 1 - 3 = 1 - 3 \Rightarrow y = -2$

$x = 2 \Rightarrow y = 2 \times 2 - 3 = 4 - 3 \Rightarrow y = 1$

$x$	-2	-1	0	1	2
$y$	1	-2	-3	-2	1

Complete the table of values.

**a)** Complete the table of values for the non-linear rule  $y = 2x^2$  (parabola).

$x = -1 \Rightarrow y = 2 \times -1 \times -1 = 2 \times 1 \Rightarrow y = 2$

$x = 0 \Rightarrow y = 2 \times 0 \times 0 = 2 \times 0 \Rightarrow y = 0$

$x = 1 \Rightarrow y = 2 \times 1 \times 1 = 2 \times 1 \Rightarrow y = 2$

$x = 2 \Rightarrow y = 2 \times 2 \times 2 = 4 \times 2 \Rightarrow y = 8$

$x$	-2	-1	0	1	2
$y$	8				
$(x,y)$	(-2,8)	( , )	( , )	( , )	( , )

**b)** Complete the table of values for the non-linear rule  $y = x^2 + 2$  (parabola).

$x = -1 \Rightarrow y = -1 \times -1 + 2 = 1 + 2 \Rightarrow y = 3$

$x = 0 \Rightarrow y = \Rightarrow$

$x = 1 \Rightarrow y = \Rightarrow$

$x = 2 \Rightarrow y = \Rightarrow$

$x$	-2	-1	0	1	2
$y$	6				
$(x,y)$	(-2,6)	( , )	( , )	( , )	( , )

**c)** Complete the table of values for the non-linear rule  $y = -\frac{1}{x}$  (hyperbola).

$x = -2 \Rightarrow y = \Rightarrow$

$x = -1 \Rightarrow y = \Rightarrow$

$x = 1 \Rightarrow y = \Rightarrow$

$x = 2 \Rightarrow y = \Rightarrow$

$x$	-2	-1	0	1	2
$y$			X		

**d)** Complete the table of values for the non-linear rule  $y = 4^x$  (exponential function).

$x = -1 \Rightarrow y = 4^{-1} \Rightarrow y = \frac{1}{4}$

$x = 0 \Rightarrow y = \Rightarrow$

$x = 1 \Rightarrow y = \Rightarrow$

$x = 2 \Rightarrow y = \Rightarrow$

$x$	-2	-1	0	1	2
$y$	$\frac{1}{16}$				

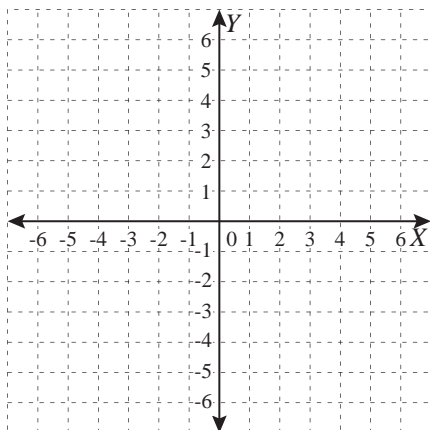
### Skill 21.15 Sketching non-linear rules by completing a table of values.

MM5.2 11 22 33 44  
MM6.1 11 22 33 44

- Complete the table of values for the equation. (see skill 21.14, page 243)
- Plot each point on the Cartesian plane.
- Draw the curved line that joins these points.
- Label the line with the rule.

**Q.** Sketch the non-linear rule  $y = x^2 - 3$  (parabola) by first completing this table of values.

$x$	-3	-2	-1	0	1	2	3
$y$	4						



**A.**  $y = x^2 - 3$

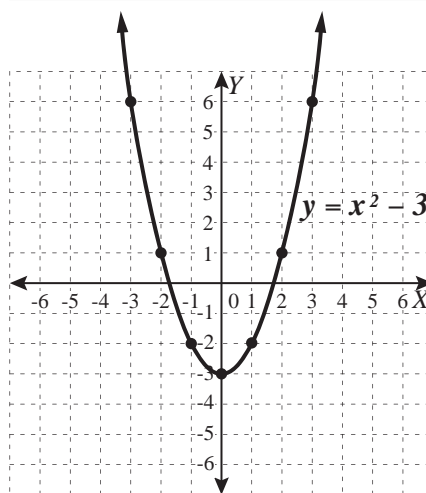
$$x = -2 \Rightarrow y = (-2)^2 - 3 = 1 \Rightarrow (-2, 1)$$

$$x = -1 \Rightarrow y = (-1)^2 - 3 = -2 \Rightarrow (-1, -2)$$

$$x = 0 \Rightarrow y = 0 - 3 = -3 \Rightarrow (0, -3)$$

$$x = 1 \Rightarrow y = 1^2 - 3 = -2 \Rightarrow (1, -2)$$

$x$	-3	-2	-1	0	1	2	3
$y$	6	1	-2	-3	-2	1	6



Complete the table of values.

Plot the points.

Join the points with a curved line.

Label the line with the rule.

**a)** Sketch the non-linear rule  $y = 2x^2$  (parabola) by first completing this table of values.

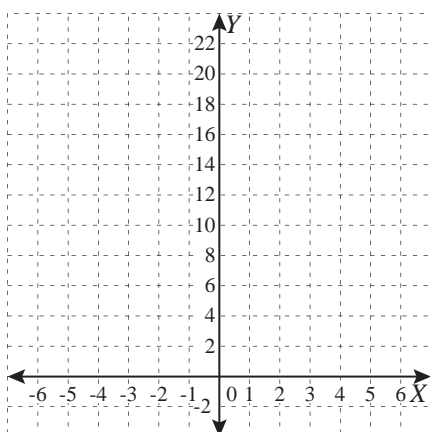
$$x = -3 \Rightarrow y = 2(-3)^2 = 18 \Rightarrow (-3, 18)$$

$$x = -2 \Rightarrow y = 2(-2)^2 = 8 \Rightarrow (-2, 8)$$

$$x = -1 \Rightarrow y = \quad \Rightarrow$$

$$x = 0 \Rightarrow y = \quad \Rightarrow$$

$x$	-3	-2	-1	0	1	2	3
$y$							



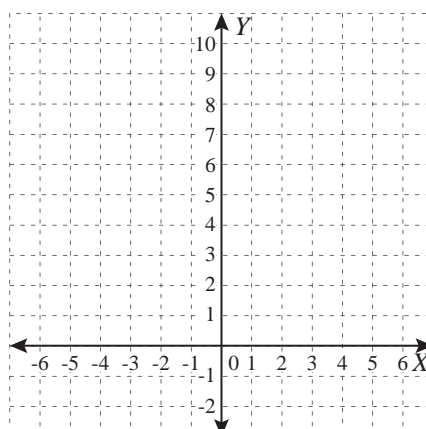
**b)** Sketch the non-linear rule  $y = x^2 - 2$  (parabola) by first completing this table of values.

.....

.....

.....

$x$	-3	-2	-1	0	1	2	3
$y$	7						



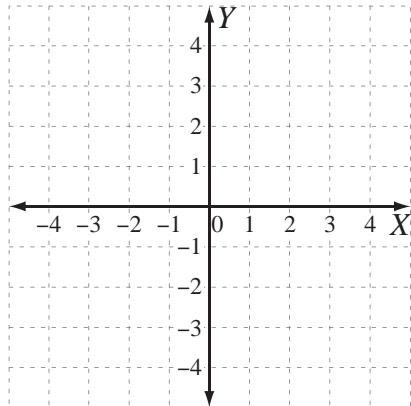
**Skill 21.16** Solving simultaneous equations by graphing their lines on a Cartesian plane (1).

- Graph the equations by finding the  $x$ -intercept and the  $y$ -intercept on the same Cartesian plane. (see skill 21.11, page 238)

OR

- Graph the equations by finding two convenient points that belong to each equation.
- Mark the solution of the simultaneous equations as the intersection point of the two lines.

**Q.** Solve the simultaneous equations  $3x + 4y = -10$  and  $5x - 2y = 18$  by graphing their lines on the Cartesian plane.



**A.** Equation 1

$$x\text{-intercept} \Rightarrow y = 0 \Rightarrow 3x = -10$$

$$3x \div 3 = -10 \div 3$$

$$x = -\frac{10}{3}$$

$$\Rightarrow x\text{-intercept is } \left(-\frac{10}{3}, 0\right)$$

$$y\text{-intercept} \Rightarrow x = 0 \Rightarrow 4y = -10$$

$$4y \div 4 = -10 \div 4$$

$$y = -\frac{5}{2}$$

$$\Rightarrow y\text{-intercept is } \left(0, -\frac{5}{2}\right)$$

Equation 2

$$x\text{-intercept} \Rightarrow y = 0 \Rightarrow 5x = 18$$

$$5x \div 5 = 18 \div 5$$

$$x = \frac{18}{5}$$

$$\Rightarrow x\text{-intercept is } \left(\frac{18}{5}, 0\right)$$

$$y\text{-intercept} \Rightarrow x = 0 \Rightarrow -2y = 18$$

$$-2y \div -2 = 18 \div -2$$

$$y = -9$$

$$\Rightarrow y\text{-intercept is } (0, -9)$$

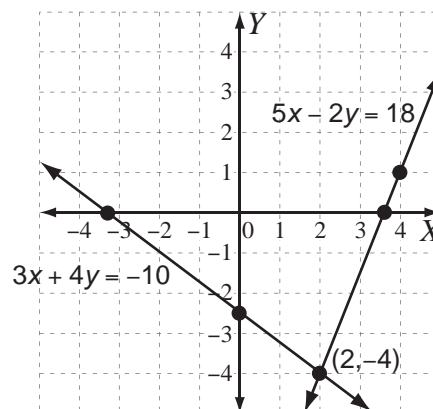
*-9 is too big, so choose a different point:*

$$x = 4 \Rightarrow 20 - 2y = 18$$

$$-2y \div -2 = -2 \div -2$$

$$y = 1$$

$$\Rightarrow \text{point } (4, 1)$$



Plot all the points.  
Sketch both graphs by joining the respective pairs of points.  
Mark the intersection.

The lines intersect at the point  $(2, -4)$   
Solution is  $(2, -4)$

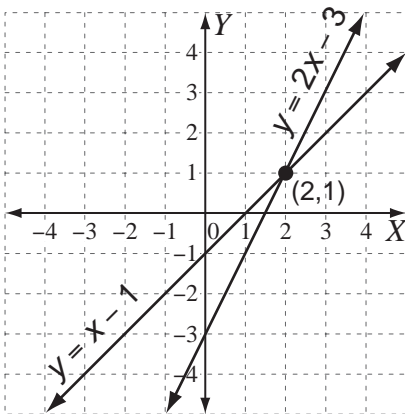
**Skill 21.16** Solving simultaneous equations by graphing their lines on a Cartesian plane (2).

MM5.2 11 22 33 44  
MM6.1 11 22 33 44

- a)** Solve the simultaneous equations  $y = x - 1$  and  $y = 2x - 3$  by graphing their lines on the Cartesian plane.

Equation 1  $y = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1 \Rightarrow (1, 0)$   
 Equation 2  $x = 0 \Rightarrow y = 0 - 1 = -1 \Rightarrow (0, -1)$

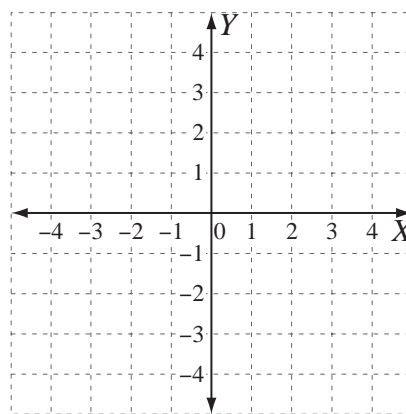
Equation 1  $y = 0 \Rightarrow 2x - 3 = 0 \Rightarrow x = \frac{3}{2} \Rightarrow (\frac{3}{2}, 0)$   
 Equation 2  $x = 0 \Rightarrow y = 0 - 3 = -3 \Rightarrow (0, -3)$



- b)** Solve the simultaneous equations  $y = 3x + 1$  and  $x + 1 = 0$  by graphing their lines on the Cartesian plane.

Equation 1  $y = 0 \Rightarrow$   
 Equation 2  $x = 0 \Rightarrow$

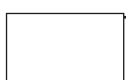
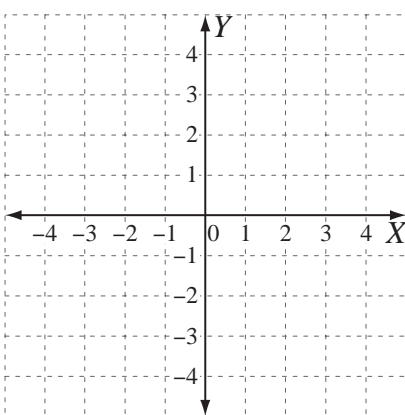
Equation 1  $\Rightarrow$   
 Equation 2  $\Rightarrow$



- c)** Solve the simultaneous equations  $x + y = 4$  and  $2x + y = 6$  by graphing their lines on the Cartesian plane.

Equation 1  $\Rightarrow$   
 Equation 2  $\Rightarrow$

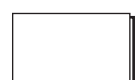
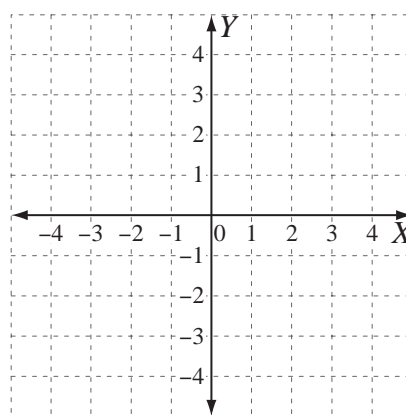
Equation 1  $\Rightarrow$   
 Equation 2  $\Rightarrow$



- d)** Solve the simultaneous equations  $2x + y = 3$  and  $x - 2y = 4$  by graphing their lines on the Cartesian plane.

Equation 1  $\Rightarrow$   
 Equation 2  $\Rightarrow$

Equation 1  $\Rightarrow$   
 Equation 2  $\Rightarrow$





### Skill 21.17 Calculating the distance between two points.

MM5.2 1 1 2 2 3 3 4 4  
MM6.1 1 1 2 2 3 3 4 4

- Identify  $(x_1, y_1)$  and  $(x_2, y_2)$  as the coordinates of the given points.
- Write the formula for the distance between two points.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Substitute the values of  $x_1$ ,  $x_2$ ,  $y_1$  and  $y_2$  into the formula.

**Q.** Use  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$  to find the distance between the points  $(-4, 1)$  and  $(-6, -3)$ . [Leave the answer in surd form.]

**A.**  $(x_1, y_1) = (-4, 1) \Rightarrow x_1 = -4$  and  $y_1 = 1$   
 $(x_2, y_2) = (-6, -3) \Rightarrow x_2 = -6$  and  $y_2 = -3$   
 $(x_2 - x_1)^2 = (-6 - (-4))^2 = (-2)^2 = 4$   
 $(y_2 - y_1)^2 = (-3 - 1)^2 = (-4)^2 = 16$   
 $d = \sqrt{4 + 16} = \sqrt{20}$

**a)** Use  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$  to find the distance between the points  $(5, 3)$  and  $(-4, 2)$ . [Leave the answer in surd form.]

$$x_1 = 5, y_1 = 3, x_2 = -4, y_2 = 2$$

$$(x_2 - x_1)^2 = (-4 - 5)^2 = (-9)^2 = 81$$

$$(y_2 - y_1)^2 = (2 - 3)^2 = (-1)^2 = 1$$

$$d = \sqrt{81 + 1} = \sqrt{82}$$

**b)** Use  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$  to find the distance between the points  $(-1, 7)$  and  $(2, -3)$ . [Leave the answer in surd form.]

$$x_1 = \quad, y_1 = \quad, x_2 = \quad, y_2 = \quad$$

$$d = \quad = \quad$$

**c)** Use  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$  to find the distance between the points  $(0, 3)$  and  $(5, -2)$ . [Leave the answer in surd form.]

$$x_1 = \quad, y_1 = \quad, x_2 = \quad, y_2 = \quad$$

$$d = \quad = \quad$$

**d)** Use  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$  to find the distance between the points  $(-4, 0)$  and  $(-3, 1)$ . [Leave the answer in surd form.]

$$x_1 = \quad, y_1 = \quad, x_2 = \quad, y_2 = \quad$$

$$d = \quad = \quad$$

**e)** Use  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$  to find the distance between the points  $(-2, -2)$  and  $(0, 4)$ . [Leave the answer in surd form.]

$$x_1 = \quad, y_1 = \quad, x_2 = \quad, y_2 = \quad$$

$$d = \quad = \quad$$

**f)** Use  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$  to find the distance between the points  $(8, 0)$  and  $(0, 8)$ . [Leave the answer in surd form.]

$$x_1 = \quad, y_1 = \quad, x_2 = \quad, y_2 = \quad$$

$$d = \quad = \quad$$

