26. [Pythagoras / Trigonometry]

Skill 26.1 Solving simple quadratic equations.

MM5.2 **1** 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Calculate the square numbers on the right-hand side of the equation.
- Evaluate and simplify the right-hand side of the equation.
- Take the square root of both sides of the equation to find the value of the unknown.
- Estimate which positive number, when multiplied by itself, produces the number under the square root.
- Check your estimation by multiplying your guess by itself.
- If the number is a decimal number consider the position of the decimal point.

Q. Find the positive solution for a:
$a^2 = 20^2 - 16^2$

A. $a^2 = 20^2 - 16^2$

$20^2 = 20 \times 20$

$16^2 = 16 \times 16$

$a^2 = 400 - 256$
$a^2 = 144$
$\sqrt{a^2} = \sqrt{144}$

$\sqrt{a^2} = a$

$a = \sqrt{12 \times 12}$
$a = \mathbf{12}$

a) Find the positive solution for c: $c^2 = 676$

$\sqrt{c^2} = \sqrt{676}$
$c = \sqrt{26 \times 26}$
$c =$ 26

b) Find the positive solution for b: $b^2 = 441$

$b =$
$b =$
$b =$

c) Find the positive solution for a: $a^2 = 225$

$a =$
$a =$
$a =$

d) Find the positive solution for b: $b^2 = 1600$

$b =$
$b =$
$b =$

e) Find the positive solution for c: $c^2 = 6.25$

$c =$
$c =$
$c =$

f) Find the positive solution for a: $a^2 = 0.16$

$a =$
$a =$
$a =$

g) Find the positive solution for c: $c^2 = 7^2 + 24^2$

$c^2 =$
$c^2 =$
$c =$
$c =$

h) Find the positive solution for a: $a^2 = 50^2 - 30^2$

$a^2 =$
$a^2 =$
$a =$
$a =$

i) Find the positive solution for b: $b^2 = 25^2 - 20^2$

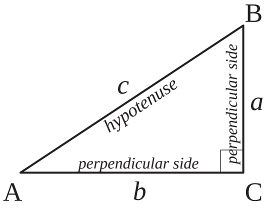
$b^2 =$
$b^2 =$
$b =$
$b =$

Skill 26.2 Recognising Pythagoras' theorem.

- Determine which is the longest side of the right-angled triangle (hypotenuse).
  Hints: In a triangle, the vertices are labelled with capital letters.
  Any side length of a triangle is usually labelled with a lower case letter (the same as the letter at the opposite vertex or angle).
- Identify correct statements of Pythagoras' theorem or ones derived from it.

Pythagoras' Theorem: $a^2 + b^2 = c^2$

For any right-angled triangle, the square of the length of the hypotenuse (c) equals the sum of the squares of the lengths of the two perpendicular sides (a and b).



OR: $c^2 = a^2 + b^2$

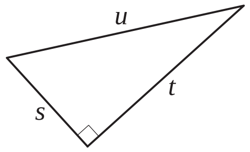
For any right-angled triangle, the square of the length of the hypotenuse (c) equals the sum of the squares of the lengths of the two perpendicular sides (a and b).

OR: $a^2 = c^2 - b^2$ **and** $b^2 = c^2 - a^2$

For any right-angled triangle, the square of the length of a perpendicular side equals the difference between the square of the length of the hypotenuse and the square of the length of the other perpendicular side.

Q. Which statements of Pythagoras' theorem are correct?

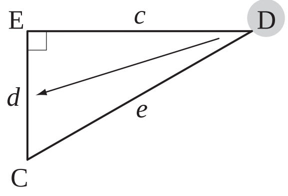
A) $t^2 + u^2 = s^2$
B) $u^2 = s^2 + t^2$
C) $s^2 = u^2 - t^2$



A. *Pythagoras' statements are:* $u^2 = s^2 + t^2$
or $s^2 + t^2 = u^2$
or $s^2 = u^2 - t^2$
or $t^2 = u^2 - s^2$

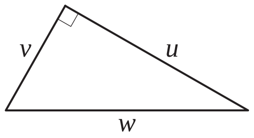
The correct statements are **B** *and* **C**.

a) Which letter marks the perpendicular side opposite to angle D in this right-angled triangle?



b) Which statements of Pythagoras' theorem are correct?

A) $w^2 = u^2 + v^2$
B) $u^2 = v^2 + w^2$
C) $v^2 = w^2 - u^2$



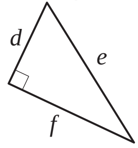
and

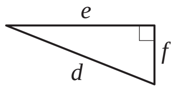
c) Connect the following Pythagoras' relationships to their corresponding diagram:

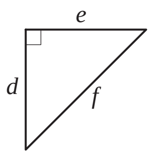
$f^2 = d^2 - e^2$

$e^2 = f^2 - d^2$

$e^2 = d^2 + f^2$





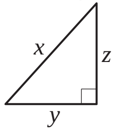


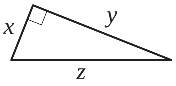
d) Connect the following Pythagoras' relationships to their corresponding diagram:

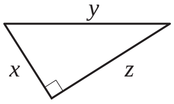
$x^2 + y^2 = z^2$

$z^2 = x^2 - y^2$

$y^2 = x^2 + z^2$







Skill 26.3 Solving more complex quadratic equations.

- Calculate the square numbers on both sides of the equation.
- Isolate the pronumeral on the left-hand side of the equation.
- Evaluate and simplify the right-hand side of the equation.
- Take the square root of both sides of the equation to find the value of the unknown.

Q. Find the positive solution for b:
$12^2 + b^2 = 15^2$

A.
$12^2 + b^2 = 15^2$
$144 + b^2 = 225$
$b^2 = 225 - 144$
$b^2 = 81$
$\sqrt{b^2} = \sqrt{81}$ — $81 = 9 \times 9$
$b = \mathbf{9}$

a) Find the positive solution for c: $12^2 + 16^2 = c^2$

$144 + 256 = c^2$
$c^2 = 400$
$\sqrt{c^2} = \sqrt{400}$
$c =$ 20

b) Find the positive solution for a: $a^2 + 15^2 = 17^2$

$a^2 +$
$a^2 =$
$\sqrt{a^2} =$
$a =$

c) Find the positive solution for b: $5^2 + b^2 = 13^2$

$25 + b^2 =$
$b^2 =$

$b =$

d) Find the positive solution for a: $a^2 + 20^2 = 25^2$

$a =$

e) Find the positive solution for b: $24^2 + b^2 = 25^2$

$b =$

f) Find the positive solution for c: $9^2 + 12^2 = c^2$

$c =$

g) Find the positive solution for c: $10^2 + 24^2 = c^2$

$c =$

h) Find the positive solution for b: $40^2 + b^2 = 50^2$

$b =$

i) Find the positive solution for c: $7^2 + 24^2 = c^2$

$c =$

Skill 26.4 Finding the hypotenuse when the other sides of a right-angled triangle are given.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

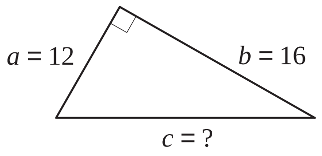
Pythagoras' Theorem

$a^2 + b^2 = c^2$

- Identify the given side lengths on the diagram.
- State Pythagoras' theorem.
- Substitute the values into Pythagoras' theorem.
- Isolate the unknown quantity on the left-hand side of the equation.
- Evaluate and simplify the right-hand side of the equation.
- Take the square root of both sides of the equation to find the value of the unknown.

Hint: The most common triplets of numbers that make Pythagoras' theorem true are:
(3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25). e.g. $3^2 + 4^2 = 5^2$ *(Pythagorean triads)*

Q. For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the hypotenuse.



A. $a = 12$ and $b = 16$

$a^2 + b^2 = c^2$

$12^2 + 16^2 = c^2$

$c^2 = 12^2 + 16^2$

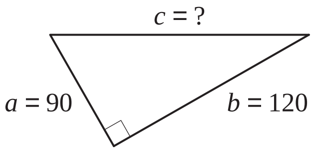
$c^2 = 144 + 256$

$c^2 = 400$

$\sqrt{c^2} = \sqrt{400}$

$c = \mathbf{20}$

a) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the hypotenuse.



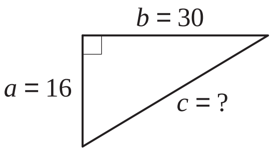
$90^2 + 120^2 = c^2$

$c^2 = 8100 + 14\,400$

$c^2 = 22\,500$

$\sqrt{c^2} = \sqrt{22\,500}$ $c =$ ☐

b) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the hypotenuse.

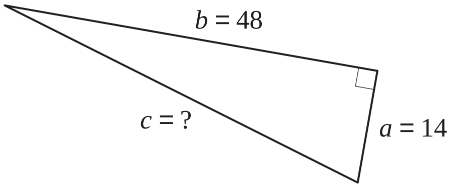


$c^2 =$

$c^2 =$

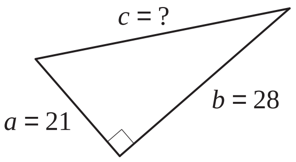
$c =$ $c =$ ☐

c) For this triangle use Pythagoras' theorem $c^2 = a^2 + b^2$. Find the length of the hypotenuse.



$c =$ $c =$ ☐

d) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the hypotenuse.



$c =$ ☐

Skill 26.5 Finding a perpendicular side when the other perpendicular side and the hypotenuse of a right-angled triangle are given.

MM5.2 1 1 2 2 **3** 3 4 4
MM6.1 **1** 1 2 2 3 3 4 4

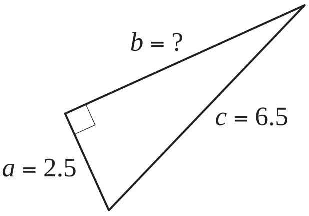
- Identify the given side lengths on the diagram.
- State Pythagoras' theorem.
- Substitute the values into Pythagoras' theorem.
- Isolate the unknown quantity on the left-hand side of the equation.
- Evaluate and simplify the right-hand side of the equation.
- Take the square root of both sides of the equation to find the value of the unknown.

Hint: The most common triplets of numbers that make Pythagoras' theorem true are:
(3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25). e.g. $3^2 + 4^2 = 5^2$ *(Pythagorean triads)*

Pythagoras' Theorem

$$a^2 + b^2 = c^2$$

Q. For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the side labelled b.



A. $a = 2.5$ and $c = 6.5$

$a^2 + b^2 = c^2$

$2.5^2 + b^2 = 6.5^2$

$b^2 = 6.5^2 - 2.5^2$

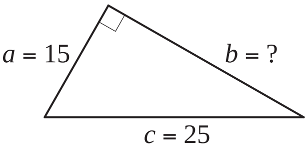
$b^2 = 42.25 - 6.25$

$b^2 = 36$

$\sqrt{b^2} = \sqrt{36}$

$b = \mathbf{6}$

a) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the side labelled b.



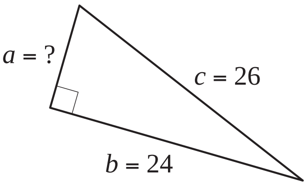
$15^2 + b^2 = 25^2$

$b^2 = 625 - 225$

$b^2 = 400$

$\sqrt{b^2} = \sqrt{400}$ $b =$ ☐

b) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the side labelled a.

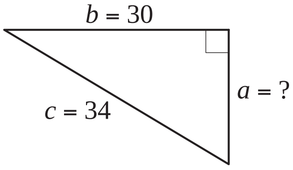


$a^2 =$

$a^2 =$

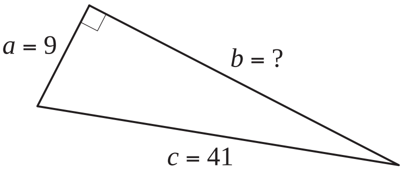
$a =$ $a =$ ☐

c) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the side labelled a.



$a =$ $a =$ ☐

d) For this triangle use Pythagoras' theorem $a^2 + b^2 = c^2$. Find the length of the side labelled b.



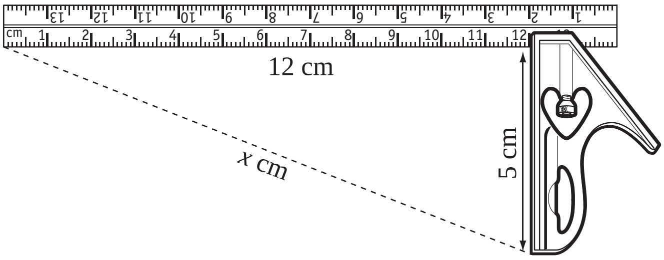
$b =$ $b =$ ☐

Skill 26.6 Applying Pythagoras' theorem (1).

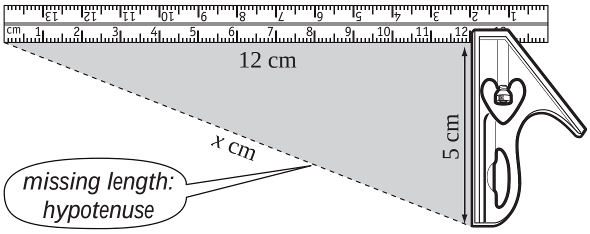
MM5.2 11 22 33 44
MM6.1 11 22 33 44

- Locate or draw a right-angled triangle in the diagram.
- Identify the given side lengths in this right-angled triangle.
- Identify the required side length in this right-angled triangle and label it with a variable.
- Use Pythagoras' theorem to find the required side length.
  (see skills 26.4, page 306 and 26.5, page 307)

Q. Find the missing length in this diagram showing a T-square.



A.



$x^2 = 12^2 + 5^2$ Pythagoras' theorem

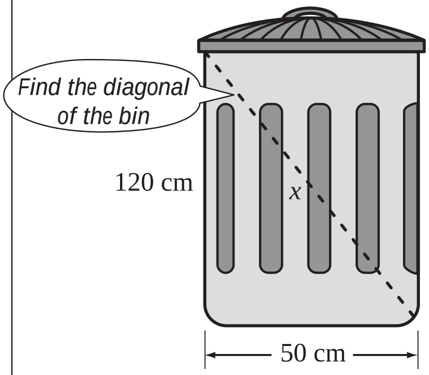
$x^2 = 144 + 25$

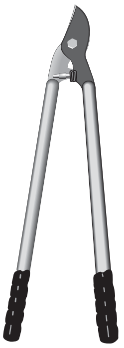
$x^2 = 169$

$x = \sqrt{169}$

$x = \mathbf{13}$

a) Would clipping shears, 125 cm long, fit inside this rubish bin with its lid on? [Objects not drawn to scale.]





$x^2 = 50^2 + 120^2$ Pythagoras' theorem

$x^2 = 2500 + 14\,400$

$x^2 = 16\,900$

$x = \sqrt{16\,900}$

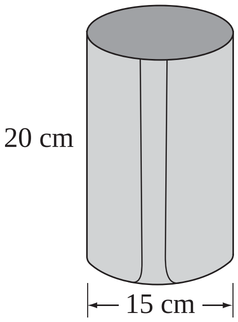
$x = 130$

clipper = 125 cm

125 cm < 130 cm

clipper fits inside the bin **yes**

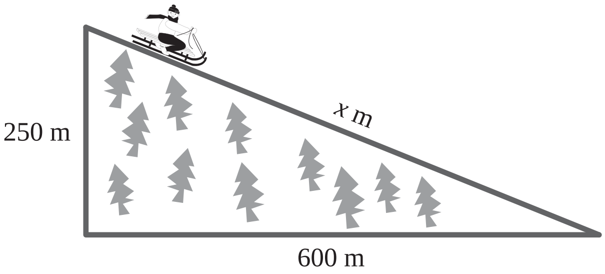
b) Would a 26 cm long paint brush fit inside this tin with its lid on? [Objects not drawn to scale.]



☐

Skill 26.6 Applying Pythagoras' theorem (2).

c) How far down this mountain slope is the sleigh descending?



$x^2 = 250^2 + 600^2$

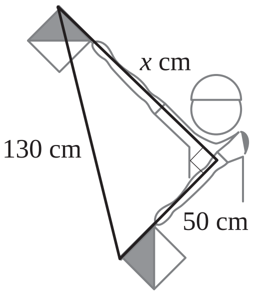
$x^2 =$

$x^2 =$

$x =$

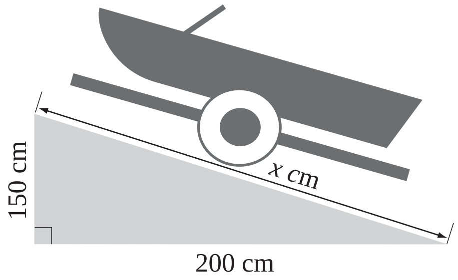
$x =$ m

d) What is the distance marked x on this diagram showing the semaphore which signals letter I?



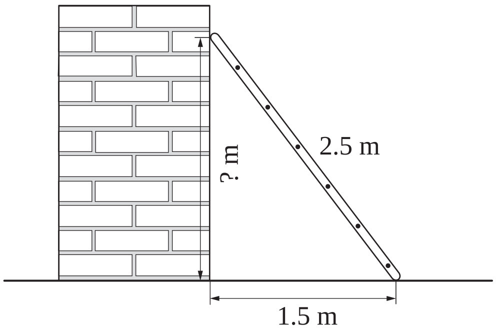
$x =$ cm

e) How long is the ramp on which the model boat descends?



$x =$ cm

f) A 2.5 m long ladder is leaning against a wall and its end is 1.5 m from the base of the wall. How high up the wall is the ladder reaching?



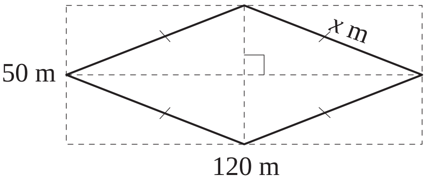
$x =$ m

Skill 26.7 Applying Pythagoras' theorem to find the perimeter of 2-dimensional shapes.

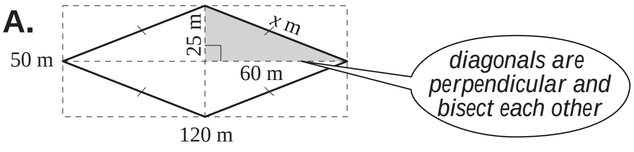
MM5.2 11 22 33 44
MM6.1 11 22 33 44

- Highlight a right-angled triangle in the diagram.
- Identify the given side lengths in this right-angled triangle.
- Identify the missing side length in this right-angled triangle and label it with a variable.
- Use Pythagoras' theorem to find the missing side length. (see skills 26.4, page 306 and 26.5, page 307)
- Calculate the perimeter of the 2-dimensional shape. (see skill 23.1, page 259)

Q. Find the perimeter of this rhombus by first calculating the missing side length.



A.



$x^2 = 25^2 + 60^2$

Pythagoras' theorem

$x^2 = 625 + 3600$

$x^2 = 4225$

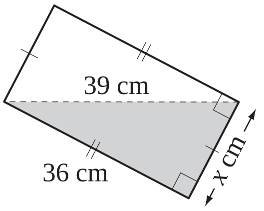
$x = \sqrt{25 \times 169}$

$x = 5 \times 13$

$x = 65$

$P = 4 \times 65$ m = **260 m**

a) Find the perimeter of this rectangle by first calculating the missing side length.



$x^2 + 36^2 = 39^2$

$x^2 = 1521 - 1296$

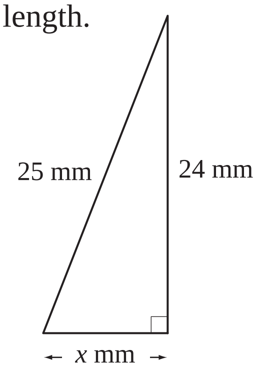
$x^2 = 225$

$x = \sqrt{225}$

$x = 15$

$P = 15 + 15 + 36 + 36$ = ______ cm

b) Find the perimeter of this triangle by first calculating the missing side length.



$x^2 + 24^2 = 25^2$

$x^2 =$ ……………

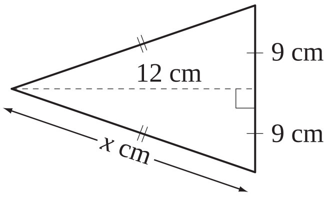
$x^2 =$ ……………

$x =$ ……………

$x =$ ……………

$P =$ …………… = ______ mm

c) Find the perimeter of this isosceles triangle by first calculating the missing side length.



……………

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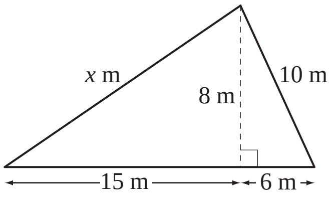
……………

……………

……………

$P =$ …………… = ______ cm

d) Find the perimeter of this triangle by first calculating the missing side length.



……………

……………

……………

……………

……………

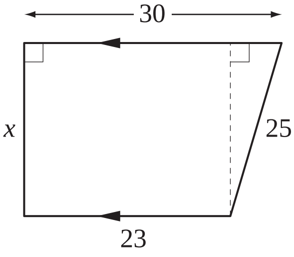
$P =$ …………… = ______ m

Skill 26.8 Applying Pythagoras' theorem in a variety of 2-dimensional shapes.

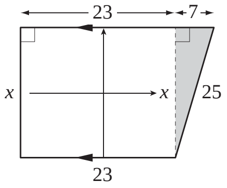
MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 **2** 2 3 3 4 4

- Highlight a right-angled triangle in the diagram.
- Identify the given lengths in this right-angled triangle.
- Identify the missing length in this right-angled triangle.
- Use Pythagoras' theorem to find the missing length. (see skills 26.4, page 306 and 26.5, page 307)

Q. Find the missing length in this trapezium.



A.



23 + 7 = 30

$x^2 + 7^2 = 25^2$ Pythagoras' theorem

$x^2 = 625 - 49$

$x^2 = 576$

$x = \sqrt{576}$

$x = \mathbf{24}$

a) Find the missing length in this rectangle.

$x^2 + 40^2 = 85^2$ Pythagoras' theorem

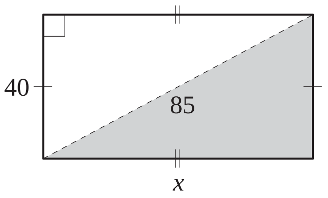
$x^2 = 7225 - 1600$

$x^2 = 5625$

$x = \sqrt{5625}$

$x = \sqrt{25 \times 225}$

$x = 5 \times 15$ = ☐



b) Find the missing length in this triangle.

$x^2 =$

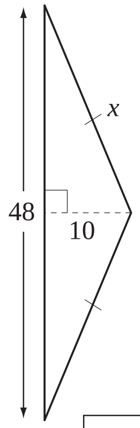
$x^2 =$

$x^2 =$

$x =$

$x =$

$x =$ = ☐



c) Find the missing length in this triangle.

$x^2 =$

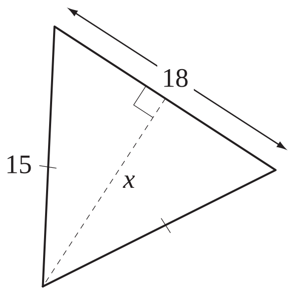
$x^2 =$

$x^2 =$

$x =$

$x =$

$x =$ = ☐



d) Find the missing length in this trapezium.

$x^2 =$

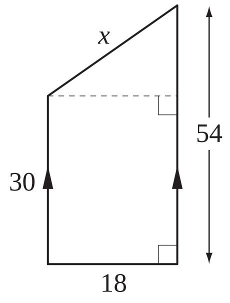
$x^2 =$

$x^2 =$

$x =$

$x =$

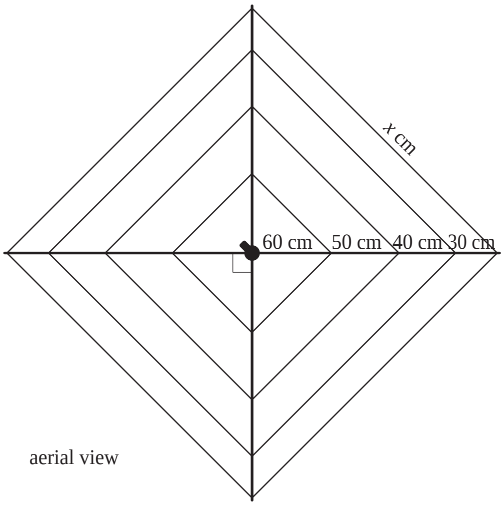
$x =$ = ☐



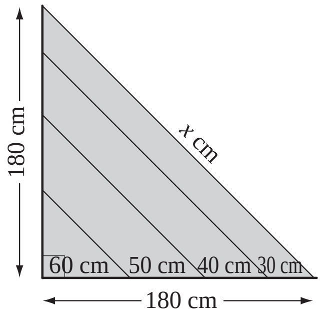
Skill 26.9 Finding a side length in isosceles right-angled triangles (1).

- Use Pythagoras' theorem in the isosceles right-angled triangle to find an unknown side length. (see skills 26.4, page 306 and 26.5, page 307)

Q. How much wire was used for the outside square of this clothes line?
[Leave your answer in surd form.]



A.



$x^2 = 180^2 + 180^2$ Pythagoras' theorem

$x^2 = 2 \times 180^2$

$x = \sqrt{2 \times 32\,400}$

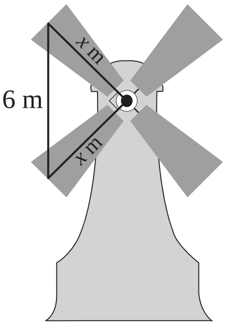
$x = \sqrt{64\,800}$

Perimeter wire $= 4x$

$= \mathbf{4 \times \sqrt{64800}}$ **cm**

(approx. 1000 cm)

a) How long is each blade of this windmill, if they are all the same length and the distance between the tips of two consecutive blades is 6 m? [Leave your answer in surd form.]



$x^2 + x^2 = 6^2$ Pythagoras' theorem

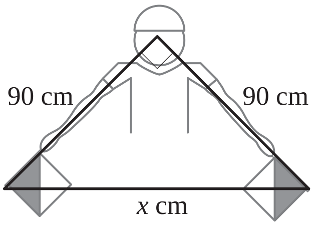
$2x^2 = 36$

$2x^2 \div 2 = 36 \div 2$

$x^2 = 18$

$x = \sqrt{18}$ .... m

b) What is the distance between the flags when this semaphore is signalling letter N as shown in the diagram? [Leave your answer in surd form.]



$x^2 = 90^2 + 90^2$

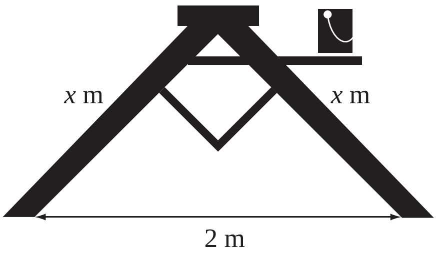
$x^2 = 2 \times$

$x =$

$x =$ .... cm

Skill 26.9 Finding a side length in isosceles right-angled triangles (2).

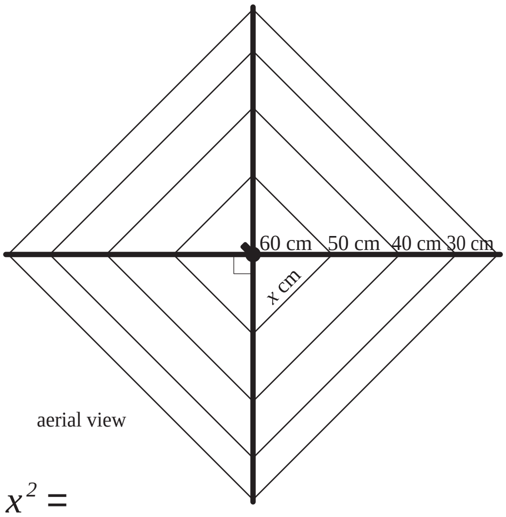
c) How long are this ladder's legs, if they are 2 m apart? [Leave your answer in surd form.]



$x^2 + x^2 = 2^2$

x = m

d) How long is the inner wire on this clothes line? [Leave your answer in surd form.]



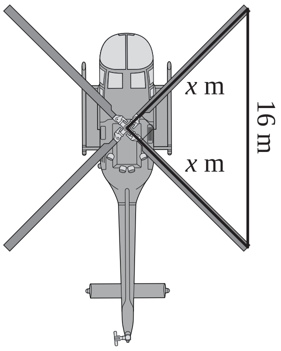
x^2 =

x^2 =

x =

x = cm

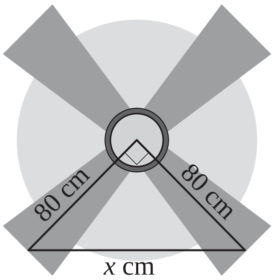
e) How long is each of these helicopter blades, if they are all the same length and the distance between the tips of two consecutive blades is 15 m? [Leave your answer in surd form.]



x^2

x = m

f) Find the missing length in this diagram showing a ceiling fan. [Leave your answer in surd form.]



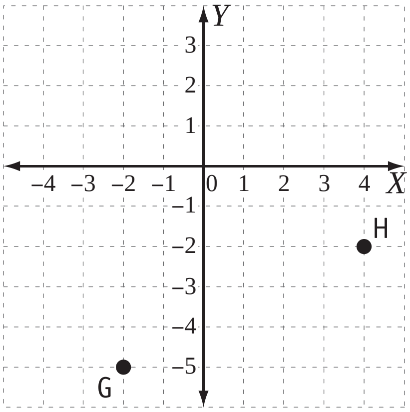
cm

Skill 26.10 Applying Pythagoras' theorem to find the distance between two points located on a Cartesian plane (1).

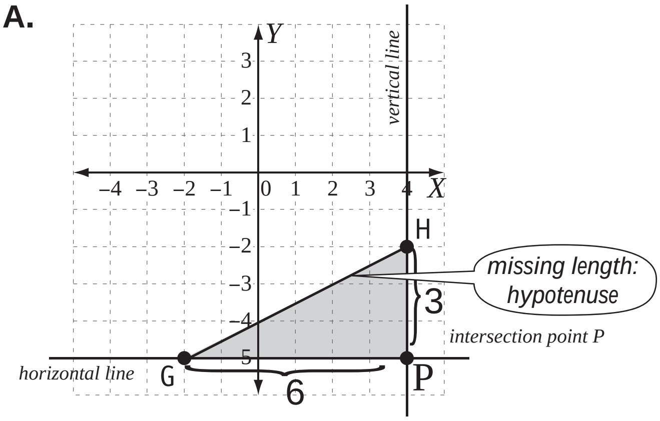
MM5.2 11 22 33 44
MM6.1 11 22 33 44

- Draw a horizontal line through the first point.
- Draw a vertical line through the second point.
- Mark the point at the intersection of these lines.
- Join the three points (the two given points and the point at the intersection) to form a triangle.
- Count the units along the horizontal and vertical sides of the triangle.
- Use Pythagoras' theorem in this right-angled triangle to find the hypotenuse.
  (see skill 26.4, page 306)

Q. Find the distance GH in this Cartesian plane.
[Leave your answer in surd form.]



A.



$GH^2 = GP^2 + PH^2$ — Pythagoras' theorem

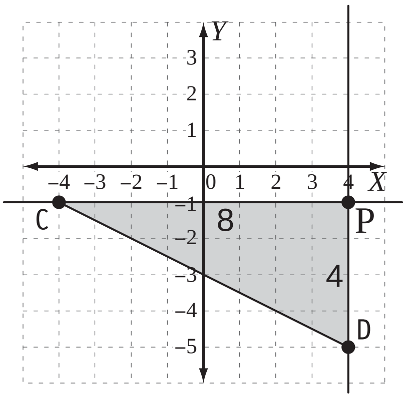
$GH^2 = 6^2 + 3^2$

$GH^2 = 36 + 9$

$GH^2 = 45$

$GH = \sqrt{\mathbf{45}}$

a) Find the distance CD in this Cartesian plane.
[Leave your answer in surd form.]



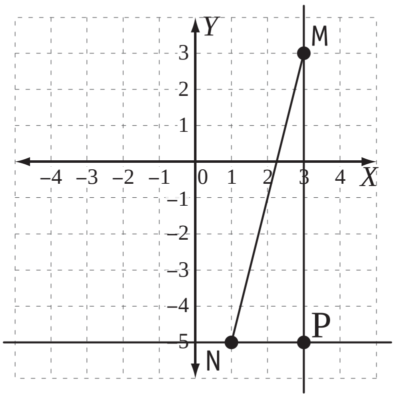
$CD^2 = CP^2 + PD^2$

$CD^2 = 8^2 + 4^2$

$CD^2 = 80$

$CD = \sqrt{80}$

b) Find the distance MN in this Cartesian plane.
[Leave your answer in surd form.]

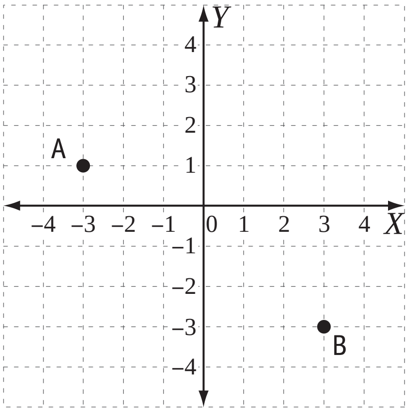


$MN^2 = MP^2 + PN^2$

Skill 26.10 Applying Pythagoras' theorem to find the distance between two points located on a Cartesian plane (2).

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

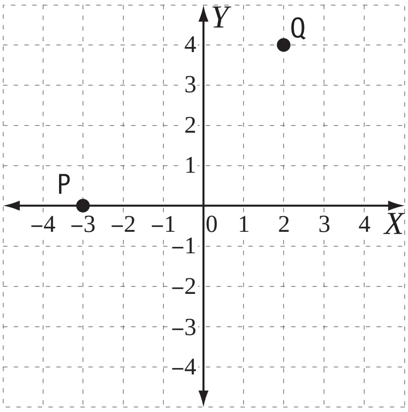
c) Find the distance AB in this Cartesian plane.
[Leave your answer in surd form.]



AB^2 =

= ☐

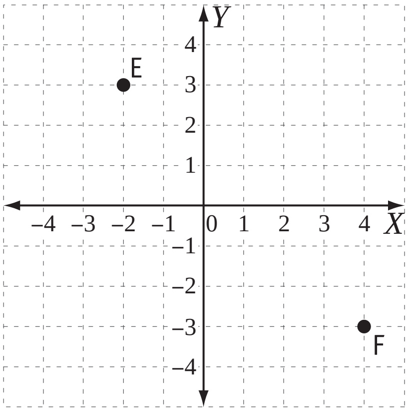
d) Find the distance PQ in this Cartesian plane.
[Leave your answer in surd form.]



PQ^2 =

= ☐

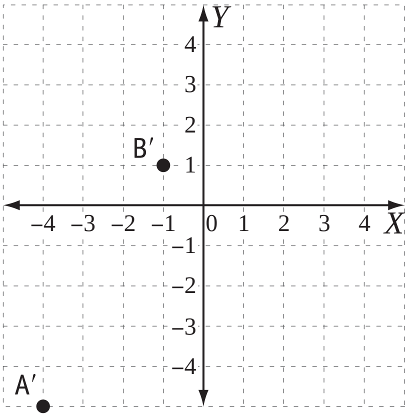
e) Find the distance EF in this Cartesian plane.
[Leave your answer in surd form.]



EF^2 =

= ☐

f) Find the distance A′B′ in this Cartesian plane.
[Leave your answer in surd form.]



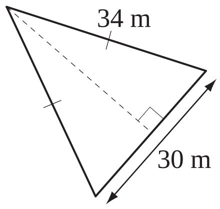
$A'B'^2$ =

= ☐

Skill 26.11 Applying Pythagoras' theorem to find the area of 2-dimensional shapes.

- Highlight a right-angled triangle in the diagram.
- Identify the given side lengths in this right-angled triangle.
- Identify the missing side length in this right-angled triangle and label it with a pronumeral.
- Use Pythagoras' theorem to find the missing side length. (see skills 26.4, page 306 and 26.5, page 307)
- Calculate the area of the 2-dimensional shape. (see skills 23.5 to 23.7, pages 265 to 267)

Q. Find the area of this triangle.



A. Let x = perpendicular height

$x^2 = 34^2 - 16^2$

$x^2 = 1156 - 256$

$x^2 = 900$

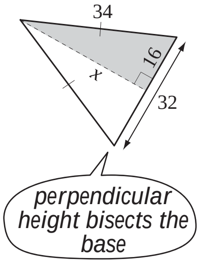
$x = \sqrt{900}$ 900 = 30 × 30

$x = 30$

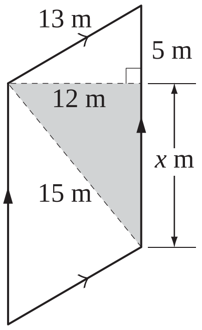
Area of triangle $= \frac{1}{2} bh$

$= \frac{1}{\cancel{2}_1} \times \overset{16}{\cancel{32}} \times 30$

$= 480$ m^2



a) Find the area of the parallelogram by first calculating the missing side length.



$x^2 + 12^2 = 15^2$

$x^2 = 225 - 144$

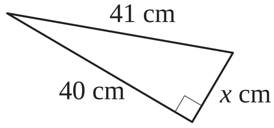
$x^2 = 81$

$x = \sqrt{81} \Rightarrow x = 9$

base$_{parallelogram}$ = 5 + 9 = 14

$A_{parall.} = bh = 14 \times 12$ = ______ m^2

b) Find the area of the right-angled triangle by first calculating the missing side length.



$x^2 + 40^2 = 41^2$

$x^2 =$

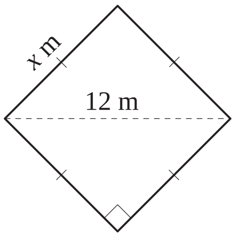
$x^2 =$

$x =$

$A_{triangle} =$

= = ______ cm^2

c) Find the area of the square by first calculating the missing side length.



$x^2 + x^2 = 12^2$

$2x^2 =$

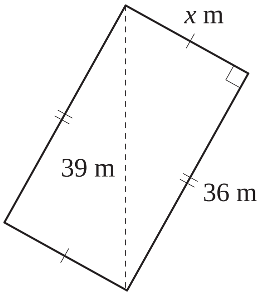
$x^2 =$

$x =$

$A_{square} =$

= = ______ m^2

d) Find the area of the rectangle by first calculating the missing side length.



$A_{rectangle} =$

= = ______ m^2

Skill 26.12 Recognising trigonometric functions (sine, cosine, tangent).

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Identify the hypotenuse of the triangle, and the opposite and adjacent sides corresponding to the marked angle (α - alpha, β - beta, θ - theta, etc).
- Label each side of the triangle with H, O and A.
- Decide which two sides of the triangle are given OR which side and angle of the triangle are given.
- Use one of the SOH - CAH - TOA relations to decide which trigonometric ratio can be used to find the unknown angle OR the unknown side.
  Hint: Use the SOH - CAH - TOA rules to remember the trigonometric ratios.

Trigonometric ratio (function) sine

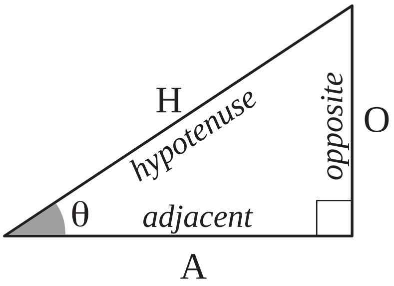
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\qquad \sin\theta = \dfrac{O}{H}$ $\qquad$ **S**ine**O**pposite**H**ypotenuse **(SOH)**

Trigonometric ratio (function) cosine

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\qquad \cos\theta = \dfrac{A}{H}$ $\qquad$ **C**osine**A**djacent**H**ypotenuse **(CAH)**

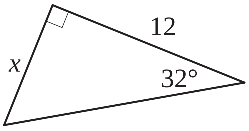
Trigonometric ratio (function) tangent

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\qquad \tan\theta = \dfrac{O}{A}$ $\qquad$ **T**angent**O**pposite**A**djacent **(TOA)**

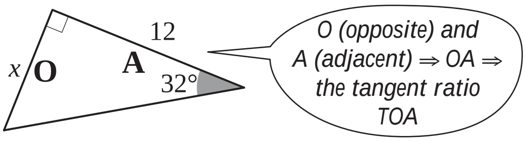


Q. Which trigonometric ratio would be used to find the unknown side *x*?

A) sine
B) cosine
C) tangent



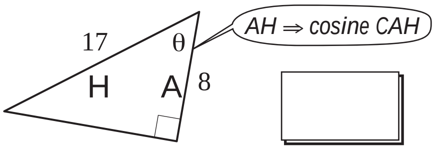
A.



The answer is ***C****.*

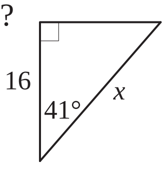
a) Which trigonometric ratio would be used to find angle θ?

A) sine
B) cosine
C) tangent



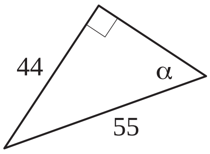
b) Which trigonometric ratio would be used to find the unknown side *x*?

A) sine
B) cosine
C) tangent



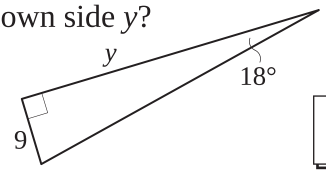
c) Which trigonometric ratio would be used to find angle α?

A) sine
B) cosine
C) tangent



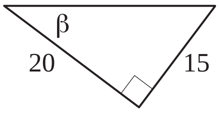
d) Which trigonometric ratio would be used to find the unknown side *y*?

A) sine
B) cosine
C) tangent



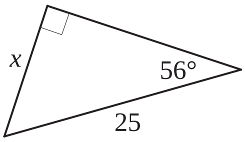
e) Which trigonometric ratio would be used to find angle β?

A) sine
B) cosine
C) tangent



f) Which trigonometric ratio would be used to find the unknown side *x*?

A) sine
B) cosine
C) tangent



Skill 26.13 Calculating the value of basic trigonometric ratios in right-angled triangles.

- Mark the angle whose trigonometric ratio is required.
- Label each side of the triangle with H, O and A.
- Use one of the SOH - CAH - TOA relations to calculate the required trigonometric ratio.

Trigonometric ratio (function) sine

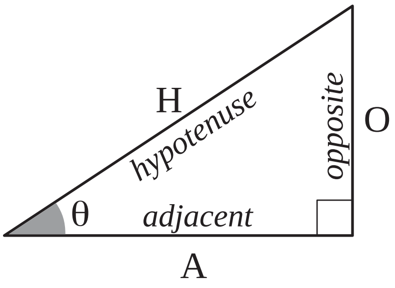
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin\theta = \frac{O}{H}$ **S**ine**O**pposite**H**ypotenuse **(SOH)**

Trigonometric ratio (function) cosine

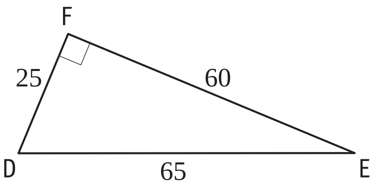
$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\cos\theta = \frac{A}{H}$ **C**osine**A**djacent**H**ypotenuse **(CAH)**

Trigonometric ratio (function) tangent

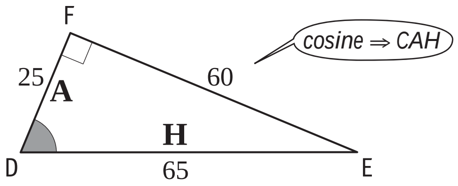
$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\tan\theta = \frac{O}{A}$ **T**angent**O**pposite**A**djacent **(TOA)**



Q. Calculate the value of cos D in this triangle.



A.

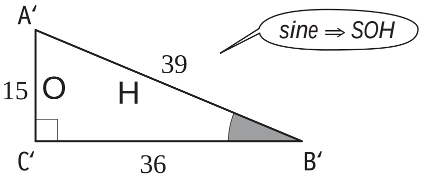


$$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{25}{65} \quad \begin{matrix}\div 5 \\ \div 5\end{matrix}$$

$$= \boldsymbol{\frac{5}{13}}$$

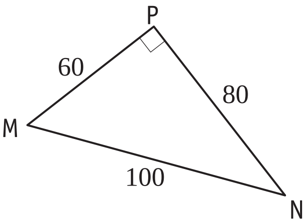
a) Calculate the value of sin B′ in this triangle.



$\sin B' \ \frac{O}{H} =$

$= \frac{15}{39} \quad \begin{matrix}\div 3 \\ \div 3\end{matrix}$ =

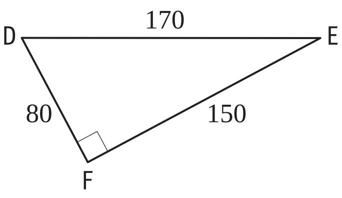
b) Calculate the value of tan M in this triangle.



$\tan M =$

= =

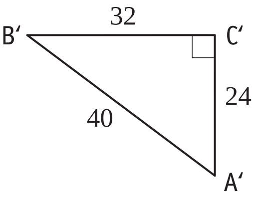
c) Calculate the value of cos E in this triangle.



$\cos E =$

= =

d) Calculate the value of sin A′ in this triangle.



$\sin A' =$

= =

Skill 26.14 Finding an unknown side of a right-angled triangle when a trigonometric ratio of an angle and another side of the triangle are given (1).

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Label each side of the triangle with H, O and A.
- Use the SOH or CAH or TOA relation corresponding to the given trigonometric ratio value.
- Substitute the numeric values in the relation.
- Solve the equation for the unknown side length.

Trigonometric ratio (function) sine

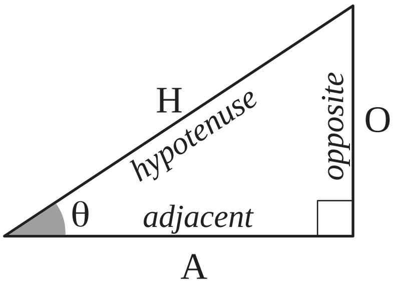
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin\theta = \frac{O}{H}$ **S**ine**O**pposite**H**ypotenuse **(SOH)**

Trigonometric ratio (function) cosine

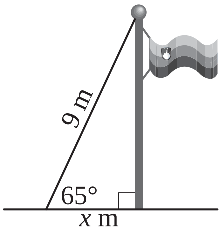
$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\cos\theta = \frac{A}{H}$ **C**osine**A**djacent**H**ypotenuse **(CAH)**

Trigonometric ratio (function) tangent

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\tan\theta = \frac{O}{A}$ **T**angent**O**pposite**A**djacent **(TOA)**



Q. A 9 m support wire is attached to a flagpole and makes an angle of 65° with the ground. If cos 65° ≈ 0.42, find the approximate distance from the end of the wire to the base of the flagpole.



A. $\cos 65° = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{H}$

$0.42 = \frac{x}{9}$ cosine ⇒ CAH

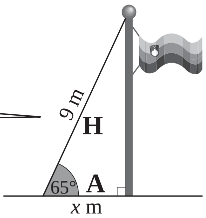
$\frac{42}{100} \times \frac{x}{9}$ Cross multiply

$42 \times 9 = 100 \times x$

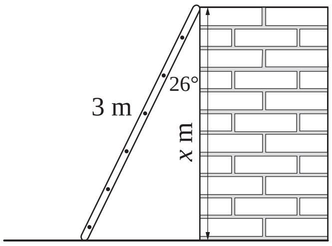
$100x = 378$

$x = 378 \div 100$

$\boldsymbol{x = 3.78\text{ m}}$



a) A 3 m ladder is leaning against a wall and makes an angle of 26° with the wall. If cos 26° ≈ 0.89, how high up the wall is the top of the ladder?



$\cos 26° = \frac{\text{adjacent}}{\text{hypotenuse}}$

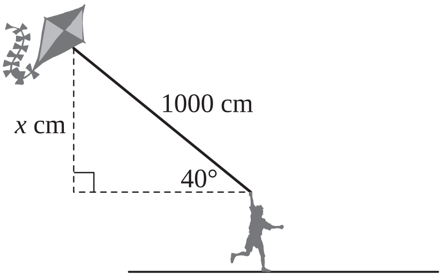
$\frac{89}{100} = \frac{x}{3}$ Cross multiply

$100x = 267$

$x = 267 \div 100$

$x =$ ☐ m

b) A kite's string makes an angle of 40° with the horizon. The string length is 1000 cm and sin 40°≈ 0.64. If the boy's height is 160 cm, how high above the ground is the kite flying?

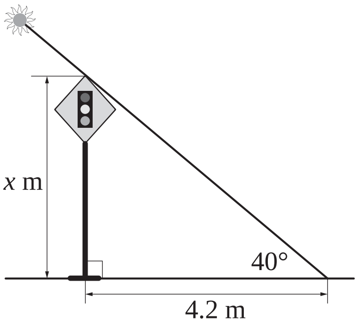


$\sin 40° =$

$x =$ height = ☐ cm

Skill 26.14 Finding an unknown side of a right-angled triangle when a trigonometric ratio of an angle and another side of the triangle are given (2).

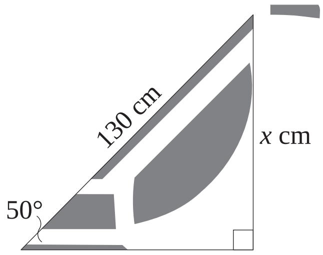
c) A road sign casts a shadow which is 4.2 m long when the sun is at an angle of 40° in the sky. If tan 40° ≈ 0.84, find the height of the road sign. [Give your answer correct to 2 decimal places.]



tan 40° =

$x =$ ☐ m

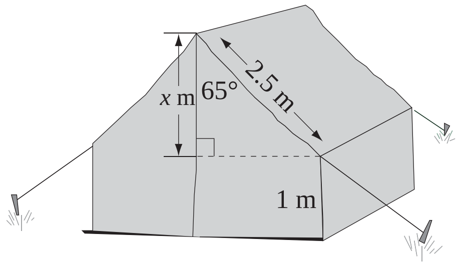
d) In this profile view the vacuum cleaner makes an angle of 50° with the ground. If sin 50°≈ 0.77, how high above the ground is the handle of the vacuum cleaner?



sin 50° =

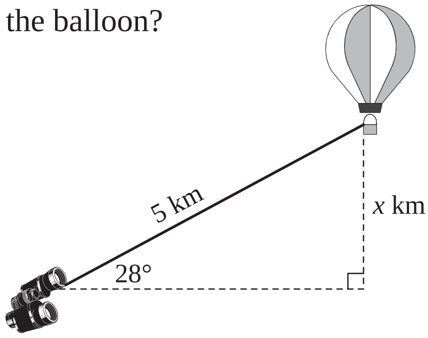
$x =$ ☐ cm

e) If cos 65° ≈ 0.42, what is the height of this tent above the ground?



$x =$ *height* = ☐ m

f) You are observing a hot air balloon which is 5 km away from you and makes an angle of 28° with your eye level. If sin 28°≈ 0.47, how high above eye level is the balloon?



$x =$ ☐ km

Skill 26.15 Calculating the value of trigonometric ratios in right-angled triangles by first applying Pythagoras' theorem (1).

MM5.2 11 22 33 44
MM6.1 11 22 33 44

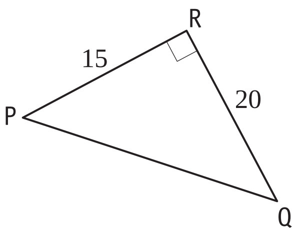
- Label each side of the triangle with H, O and A.
- Apply Pythagoras' theorem to calculate the unknown side length of the triangle. (see skills 26.4, page 306 and 26.5, page 307)
- Use one of the SOH - CAH - TOA relations to calculate the required trigonometric ratio.

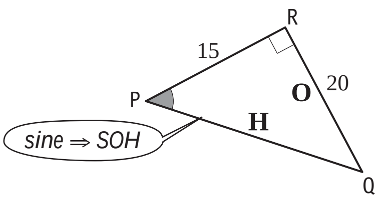
Pythagoras' Theorem

$$a^2 + b^2 = c^2$$

$\sin\theta = \frac{O}{H}$ **(SOH)**	$\cos\theta = \frac{A}{H}$ **(CAH)**	$\tan\theta = \frac{O}{A}$ **(TOA)**

Q. Calculate the value of sin P in this triangle.
[Hint: Pythagoras' theorem will help.]





A. $\sin P = \frac{opposite}{hypotenuse} = \frac{RQ}{PQ}$ unknown PQ

$PQ^2 = PR^2 + RQ^2$ Pythagoras

$PQ^2 = 15^2 + 20^2$

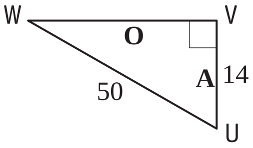
$PQ^2 = 225 + 400$

$PQ^2 = 625$

$PQ = 25$

$\sin P = \frac{RQ}{PQ} = \frac{20}{25} = \mathbf{\frac{4}{5}}$

a) Calculate the value of tan U in this triangle.
[Hint: Pythagoras' theorem will help.]



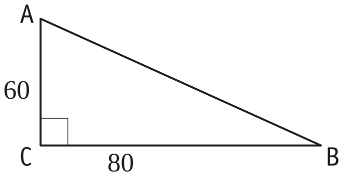
$\tan U = \frac{opposite}{adjacent} = \frac{VW}{UV}$ tangent ⇒ TOA

$VW^2 = UW^2 - UV^2$

$VW^2 = 2500 - 196 = 2304 \Rightarrow VW = 48$

$\tan U = \frac{VW}{UV} = \frac{48}{14}$ $=$ ☐

b) Calculate the value of sin B in this triangle.
[Hint: Pythagoras' theorem will help.]



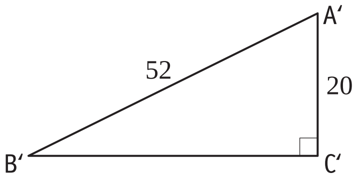
$\sin B =$

$AB^2 =$

$AB^2 =$ $\Rightarrow AB =$

$\sin B =$ $=$ ☐

c) Calculate the value of cos B′ in this triangle.
[Hint: Pythagoras' theorem will help.]



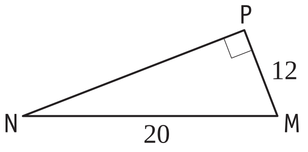
$\cos B' =$

$B'C'^2 =$

$B'C'^2 =$ $\Rightarrow B'C' =$

$\cos B' =$ $=$ ☐

d) Calculate the value of cos N in this triangle.
[Hint: Pythagoras' theorem will help.]



$\cos N =$

$NP^2 =$

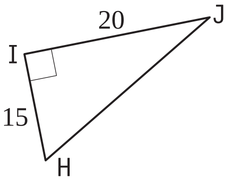
$NP^2 =$ $\Rightarrow NP =$

$\cos N =$ $=$ ☐

Skill 26.15 Calculating the value of trigonometric ratios in right-angled triangles by first applying Pythagoras' theorem (2).

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 **4**

e) Calculate the value of cos J in this triangle.
[Hint: Pythagoras' theorem will help.]



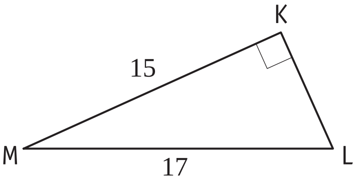
$\cos J =$

$HJ^2 =$

$HJ^2 =$ $\Rightarrow HJ =$

$\cos J =$ $=$

f) Calculate the value of cos L in this triangle.
[Hint: Pythagoras' theorem will help.]



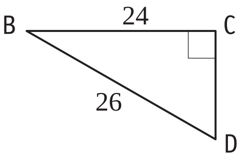
$\cos L =$

$KL^2 =$

$KL^2 =$ $\Rightarrow KL =$

$\cos L =$ $=$

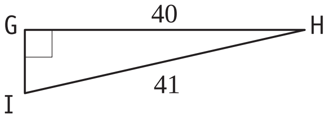
g) Calculate the value of tan D in this triangle.
[Hint: Pythagoras' theorem will help.]



$\tan D =$

$=$

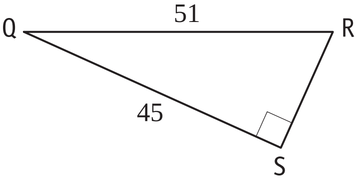
h) Calculate the value of tan H in this triangle.
[Hint: Pythagoras' theorem will help.]



$\tan H =$

$=$

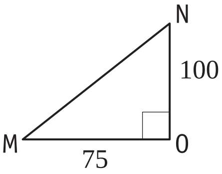
i) Calculate the value of sin Q in this triangle.
[Hint: Pythagoras' theorem will help.]



$\sin Q =$

$=$

j) Calculate the value of sin M in this triangle.
[Hint: Pythagoras' theorem will help.]



$\sin M =$

$=$