

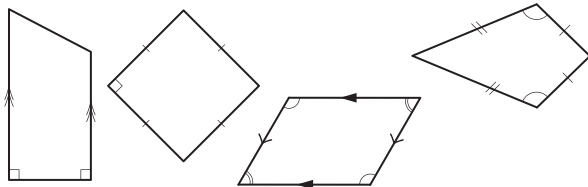
28. [Geometric Reasoning]

Skill 28.1 Recognising polygons, quadrilaterals and triangles.

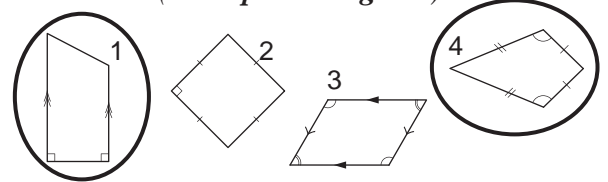
MM5.2 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Consider the definition of a polygon. (see Glossary, page 423)
- Consider the properties of a parallelogram:
 - both pairs of opposite sides are parallel and equal in length.
- Consider the properties of an isosceles triangle:
 - two sides and two corresponding angles are equal.

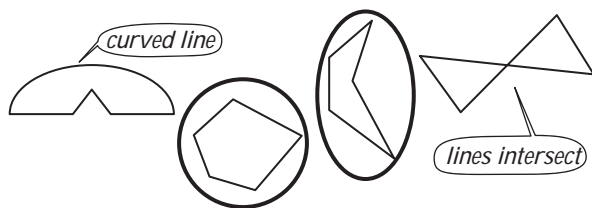
Q. Circle the shapes that are **not** parallelograms.



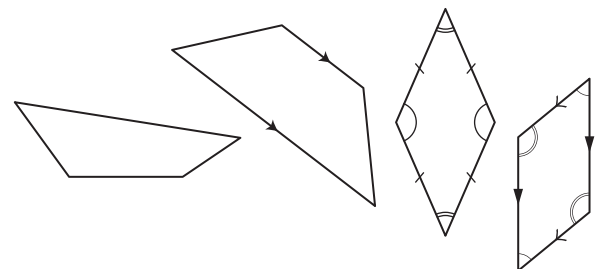
- A.** 1st shape - has only two opposite sides parallel (**not a parallelogram**).
 2nd shape - has both pairs of opposite sides equal in length (parallelogram).
 3rd shape - has both pairs of opposite sides parallel (parallelogram).
 4th shape - doesn't have any parallel sides (**not a parallelogram**).



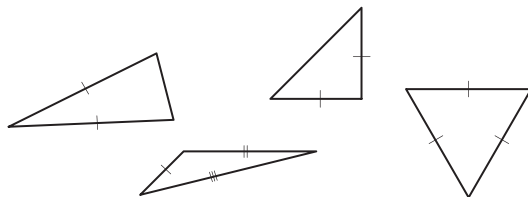
a) Circle the shapes that are polygons.



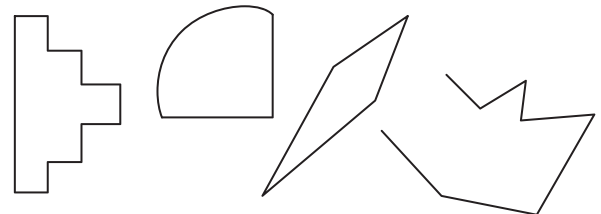
b) Circle the shapes that are **not** parallelograms.



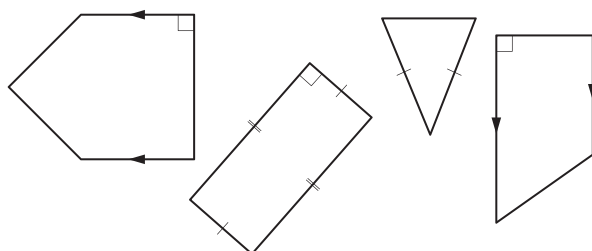
c) Circle the shape that is **not** an isosceles triangle.



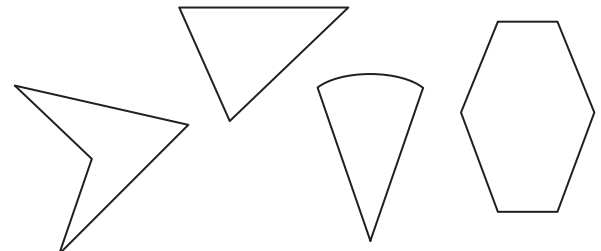
d) Circle the shapes that are **not** polygons.



e) Circle the shape that is a parallelogram.



f) Circle the shape that is **not** a polygon.



Skill 28.2 Classifying triangles.

MM5.2 1 1 22 33 44
MM6.1 1 1 22 33 44

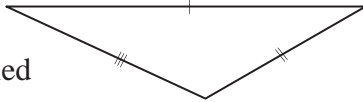
- Look for equal sides or equal angles.
- Look at the types of angles inside the triangle.

Sides and angles	Triangle type
no equal sides/angles	scalene
two equal sides/angles	isosceles
three equal sides/angles	equilateral

Angles	Triangle type
all acute angles	acute-angled
one right angle	right-angled
one obtuse angle	obtuse-angled

Q. Which two options describe this triangle?

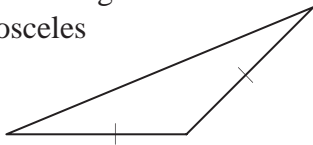
- A) scalene
B) equilateral
C) obtuse-angled



- A.** A) *scalene (no equal sides/angles)* \Rightarrow true
B) *equilateral (all equal sides)* \Rightarrow false
C) *obtuse-angled (1 obtuse angle)* \Rightarrow true
The answer is **A and C**.

a) Which two options describe this triangle?

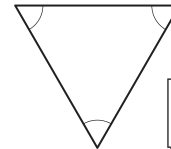
- A) right-angled
B) obtuse-angled
C) isosceles



B and C

b) Which two options describe this triangle?

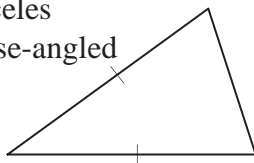
- A) equilateral
B) scalene
C) acute-angled



and

c) Which two options describe this triangle?

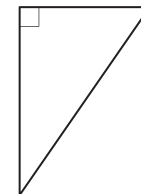
- A) acute-angled
B) isosceles
C) obtuse-angled



and

d) Which two options describe this triangle?

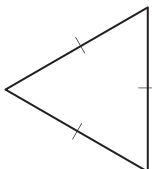
- A) acute-angled
B) right-angled
C) scalene



and

e) Which two options describe this triangle?

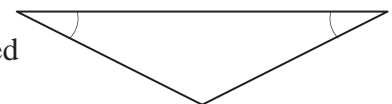
- A) acute-angled
B) scalene
C) equilateral



and

f) Which two options describe this triangle?

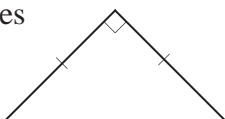
- A) isosceles
B) obtuse-angled
C) right-angled



and

g) Which two options describe this triangle?

- A) acute-angled
B) right-angled
C) isosceles



and

h) Which two options describe this triangle?

- A) scalene
B) isosceles
C) acute-angled



and

- Consider the properties of squares, rectangles, rhombi, parallelograms, kites and trapeziums. (see Glossary, page 428)

Q. I am a quadrilateral whose diagonals are not equal in length and bisect each other at right angles. What am I?
 A) square
 B) parallelogram
 C) rhombus
 D) kite

A. A) *diagonals are equal* \Rightarrow A false
 B) *diagonals do not bisect each other at right angles* \Rightarrow B false
 C) *diagonals are not equal and bisect each other at right angles* \Rightarrow C true
 D) *diagonals do not bisect each other* \Rightarrow D false

The answer is **C**.

a) I am a two-dimensional shape with 4 sides. My diagonals are not equal in length and bisect each other but not at right angles. What am I?
 A) rhombus
 B) parallelogram
 C) kite
 D) trapezium

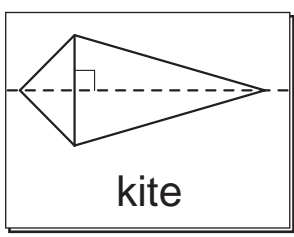
B

b) I am a quadrilateral with both pairs of opposite sides parallel and diagonals equal in length. What am I?
 A) rhombus
 B) trapezium
 C) parallelogram
 D) rectangle

c) I am a two-dimensional shape with 4 sides. Adjacent angles are not equal and I have two axes of symmetry. What am I?
 A) trapezium
 B) kite
 C) rhombus
 D) rectangle

d) I am a quadrilateral with both pairs of opposite sides equal in length, but no axis of symmetry. What am I?
 A) square
 B) trapezium
 C) parallelogram
 D) rhombus

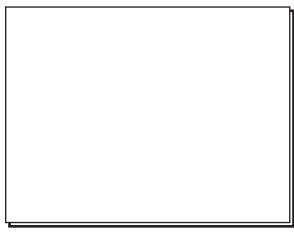
e) Draw and name the quadrilateral whose diagonals are perpendicular, but has only one axis of symmetry.



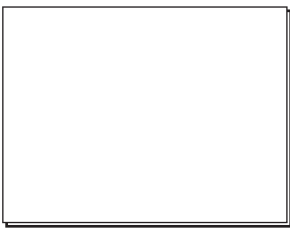
f) Draw and name the quadrilateral whose diagonals are equal in length and bisect each other at right angles.



g) Draw and name the quadrilateral whose pairs of opposite angles are equal, but does not have any axis of symmetry.



h) Draw and name the quadrilateral whose diagonals bisect each other at right angles and has two axes of symmetry.



Skill 28.4 Recognising rotational symmetry in 2-dimensional shapes.

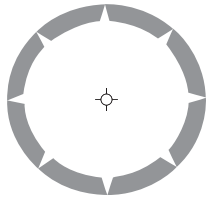
MM5.2 1 1 22 33 44
MM6.1 1 1 22 33 44

- Try to visualise the shape during a full turn of 360° .
- Count how many times during the full turn the image of the shape exactly covers the original shape.
- This number is called the order of rotational symmetry.

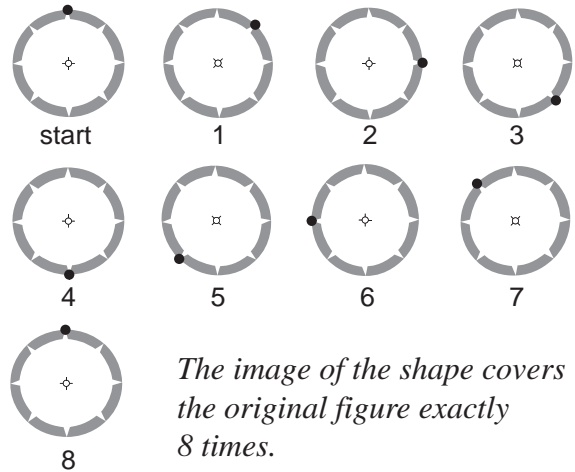
Hints: A shape does not have rotational symmetry if, during a full rotation of 360° , the image of the shape does not exactly cover the original shape.

To count how many times the image of the shape exactly covers the original shape, mark a point on the shape, so you know when the shape has done a complete rotation of 360° .

Q. What is the order of rotational symmetry for this shape?



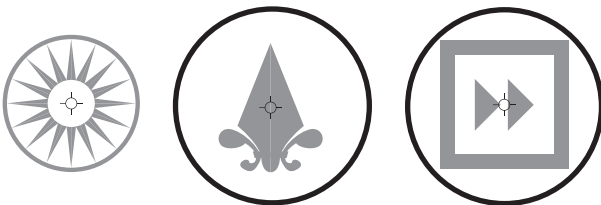
A.



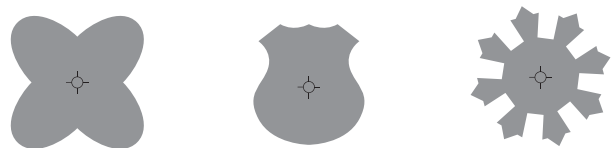
The image of the shape covers the original figure exactly 8 times.

The order of rotational symmetry = 8

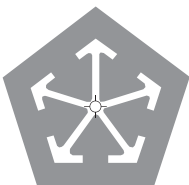
a) Circle the shapes that do not have rotational symmetry.



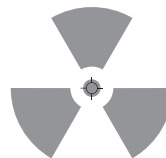
b) Circle the shapes that have rotational symmetry.



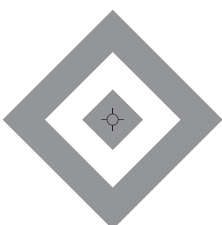
c) This shape has rotational symmetry. How many times during a full turn (360°) does the image of the shape exactly cover the original figure (order of rotational symmetry)?



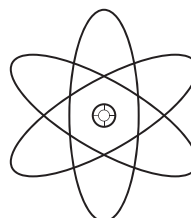
d) This shape has rotational symmetry. How many times during a full turn (360°) does the image of the shape exactly cover the original figure (order of rotational symmetry)?



e) What is the order of rotational symmetry for this shape?



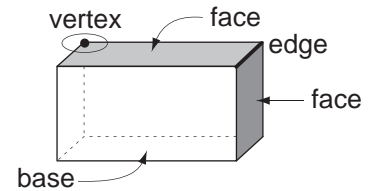
f) What is the order of rotational symmetry for this shape?



Skill 28.5 Describing the properties of 3-dimensional shapes.

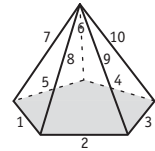
MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Count the number of:
 - faces
 - edges
 - vertices (points/corners)

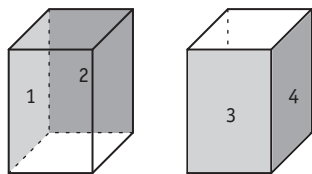


Q. How many edges does a pentagonal pyramid have?

A. *Count the number of edges, or straight lines in the pyramid: five edges in the base and five vertical edges.*
The answer is 10



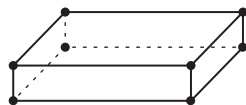
a) Of the 6 faces of a square prism, how many are rectangles?



4

b) Of the 7 faces of a pentagonal prism, how many are rectangles?

c) How many vertices does a rectangular prism have?



d) How many vertices does a triangular prism have?

e) How many edges does a tetrahedron have?

f) How many edges does a rectangular pyramid have?

g) How many faces does a pentagonal pyramid have?

h) How many faces does a triangular prism have?

i) Sketch and name the three-dimensional shape that has 6 faces, all of which are squares.



j) Sketch and name the three-dimensional shape that has 4 faces, all of which are triangles.



k) Sketch and name the three-dimensional shape that has 6 faces, five of which are triangles.



l) Sketch and name the three-dimensional shape that has 6 faces, all of which are rectangles.

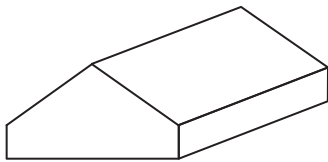


Euler's formula for any polyhedra:

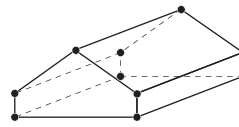
Edges = Vertices + Faces - 2 OR **E = V + F - 2**

Q. Euler's formula, $E = V + F - 2$ defines the relationship between **Edges**, **Vertices** and **Faces** of any polyhedron. Verify Euler's formula for this solid:

= + - 2



A.



$E = 15$

$V = 10$

$F = 7$

$E = V + F - 2$

$15 = 10 + 7 - 2$

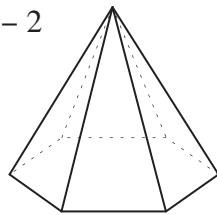
$15 = 15$ (true)

Substitute 15, 10, 7 into Euler's formula

= + - 2

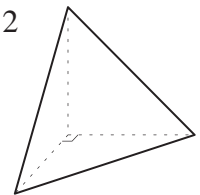
a) Euler's formula, $E = V + F - 2$ defines the relationship between **Edges**, **Vertices** and **Faces** of any polyhedron. Verify Euler's formula for a hexagonal pyramid:

= + - 2



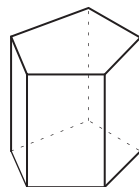
b) Euler's formula, $E = V + F - 2$ defines the relationship between **Edges**, **Vertices** and **Faces** of any polyhedron. Verify Euler's formula for a triangular pyramid:

= + - 2



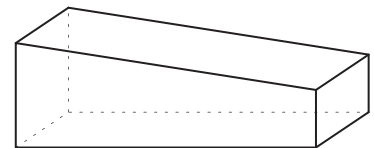
c) Euler's formula, $E = V + F - 2$ defines the relationship between **Edges**, **Vertices** and **Faces** of any polyhedron. Verify Euler's formula for a pentagonal prism:

= + - 2



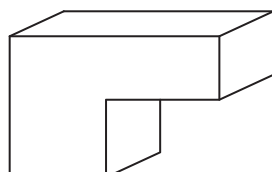
d) Euler's formula, $E = V + F - 2$ defines the relationship between **Edges**, **Vertices** and **Faces** of any polyhedron. Verify Euler's formula for this prism:

= + - 2



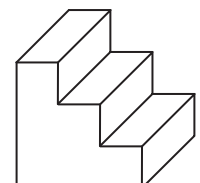
e) Euler's formula, $E = V + F - 2$ defines the relationship between **Edges**, **Vertices** and **Faces** of any polyhedron. Verify Euler's formula for this prism:

= + - 2



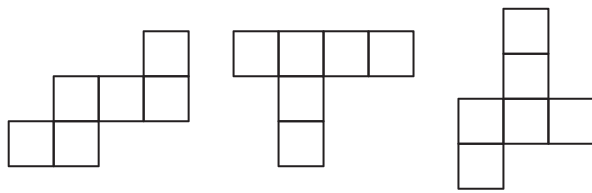
f) Euler's formula, $E = V + F - 2$ defines the relationship between **Edges**, **Vertices** and **Faces** of any polyhedron. Verify Euler's formula for this prism:

= + - 2



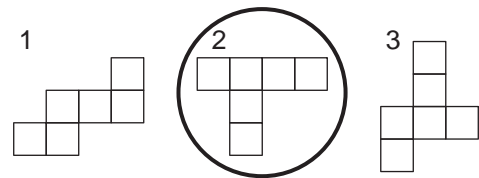
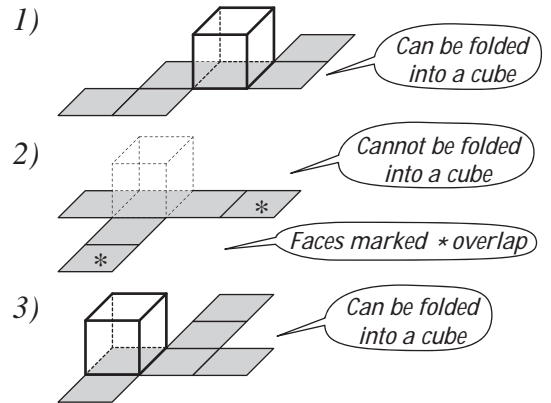
- Identify the shapes in the net.
- Imagine the shape folded. OR Make a model by tracing, cutting out and folding the net.

Q. Circle the net that **cannot** be folded to make a model of a cube.

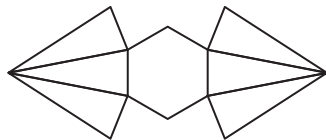


A. Enlarge, trace and cut out the shape, folding to try to form a cube.

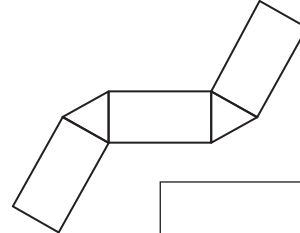
OR Imagine the shape folded:



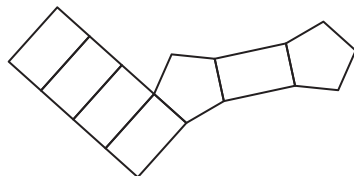
a) What three-dimensional shape can this net be used to make?



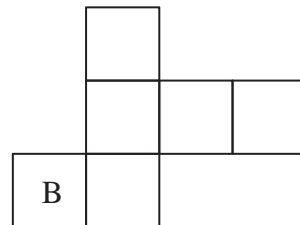
b) What three-dimensional shape can this net be used to make?



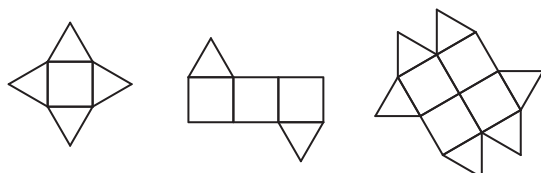
c) What three-dimensional shape can this net be used to make?



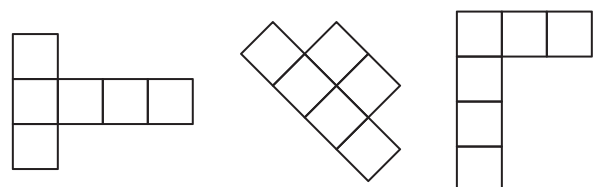
d) On this net of a cube, a face is marked B. Label the opposite face with a T.



e) Circle the net that **cannot** be folded to make a model of a three-dimensional shape.



f) Circle the net that **can** be folded to make a model of a cube.



To draw a shape translated on a grid

- Move it up (positive, vertically), down (negative, vertically), left (negative, horizontally) or right (positive, horizontally) on the grid, without flipping, turning or changing its size.

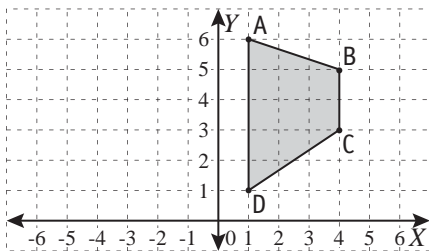
To draw a shape reflected in a given line (mirror line)

- Draw a perpendicular line to the mirror line from each vertex of the shape.
- Extend the perpendicular line beyond the mirror line by the same distance.
- Plot and join the reflected points.

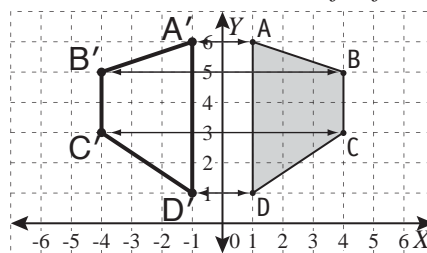
To draw a shape rotated by a given angle about a point (centre of rotation)

- Rotate each vertex by the given angle, in the given direction.
- Plot and join the rotated points.

Q. Draw the reflection of the trapezium ABCD in the line of equation $x = 0$.

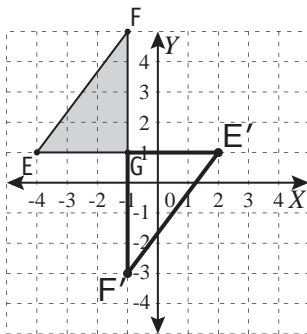


A. Reflect A, B, C and D in the mirror line $x = 0$:
A is one unit to the right of line $x = 0$
 \Rightarrow draw A' one unit to the left of line $x = 0$.

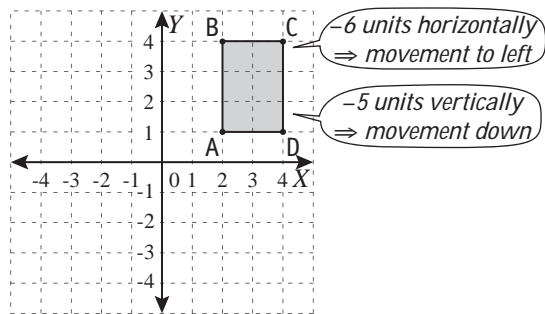


$A'B'C'D'$ is the reflection of ABCD in the line $x = 0$

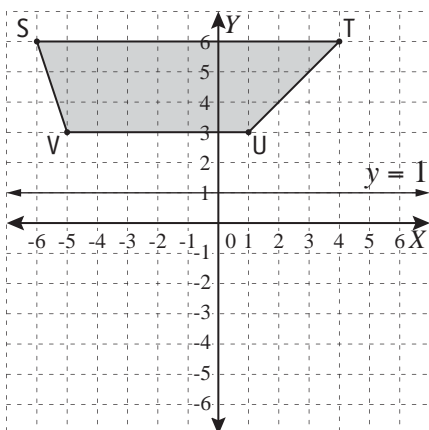
a) Redraw the triangle EFG after rotating it 180° about the point of coordinates $(-1, 2)$.



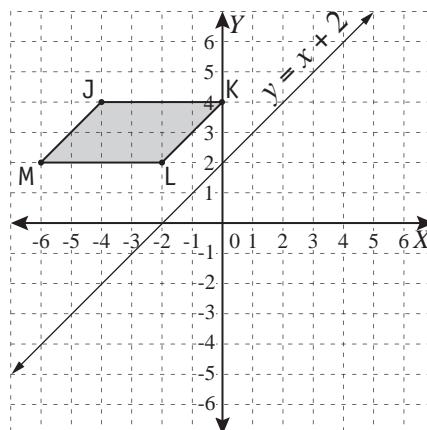
b) Redraw the rectangle ABCD after translating it -6 units horizontally and -5 units vertically.



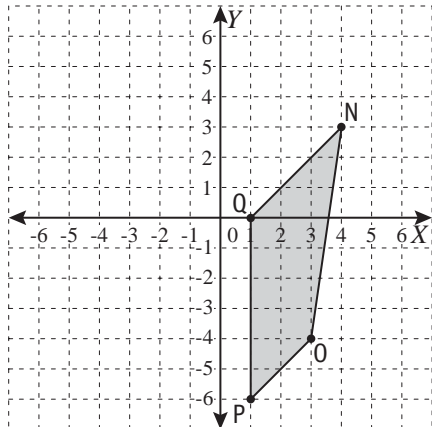
c) Draw the reflection of the trapezium STUV in the line of equation $y = 1$.



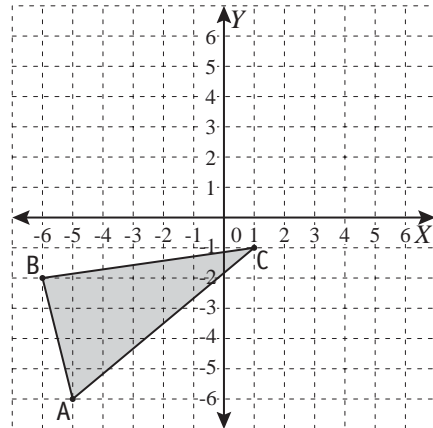
d) Draw the reflection of the parallelogram JKLM in the line of equation $y = x + 2$.



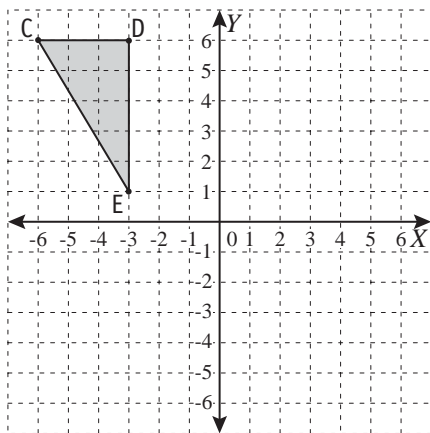
- e)** Redraw the quadrilateral NOPQ after rotating it 180° about the point of coordinates (1,0).



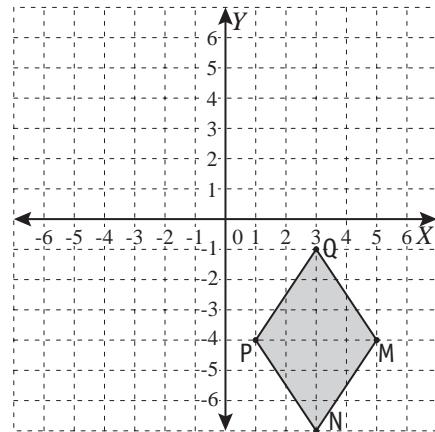
- f)** Redraw the triangle ABC after reflecting it in the x -axis and then translating it 5 units horizontally.



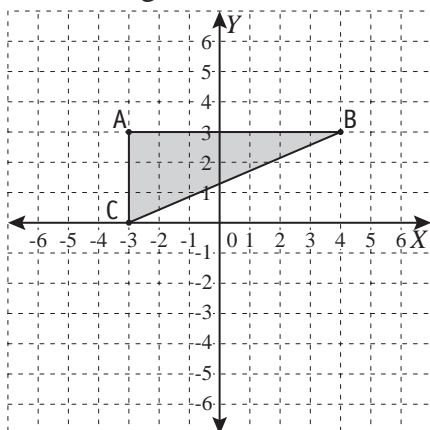
- g)** Redraw the triangle CDE after rotating it 90° clockwise about the point of coordinates $(-3, 1)$ and then translating it 4 units horizontally and -5 units vertically.



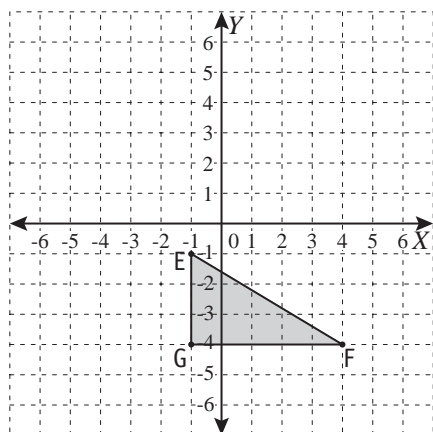
- h)** Redraw the kite MNPQ after rotating it 180° about the point of coordinates $(3, -1)$ and then reflecting it in the y -axis.



- i)** Redraw the triangle ABC after rotating it 90° anticlockwise about the point of coordinates $(-3, 0)$ and then translating it 9 units horizontally and -5 units vertically. Label the transformation $A'B'C'$. Are triangles ABC and $A'B'C'$ congruent or similar?



- j)** Redraw the triangle EFG after rotating it 180° about the point of coordinates $(-1, -1)$ and then reflecting it in the y -axis. Label the transformation $E'F'G'$. Are triangles EFG and $E'F'G'$ congruent or similar?

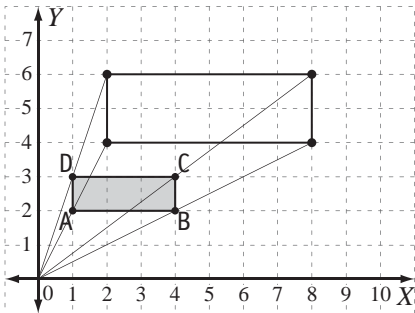


Skill 28.9 Recognising and drawing enlargements and reductions on a Cartesian plane.

MM5.2 11 22 33 44
MM6.1 11 22 33 44

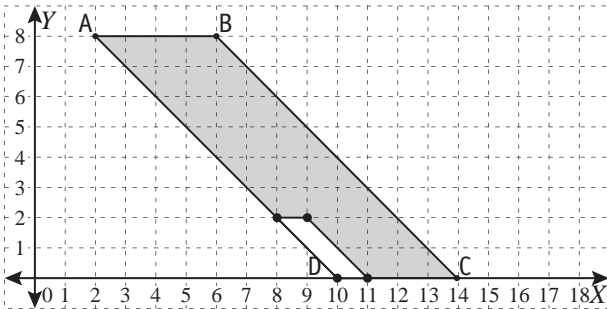
- Use the definitions of enlargement, reduction, factor of enlargement and factor of reduction. (see Glossary, pages 400, 430 and 403 respectively)

Q. Find the scale factor of enlargement for rectangle ABCD.

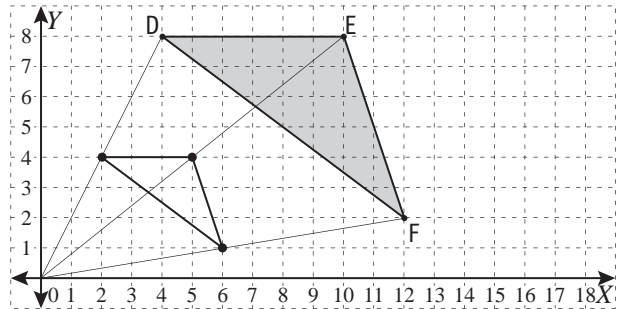


A. *The length and width of the rectangle ABCD have doubled in the enlargement.
Scale factor of enlargement = 2*

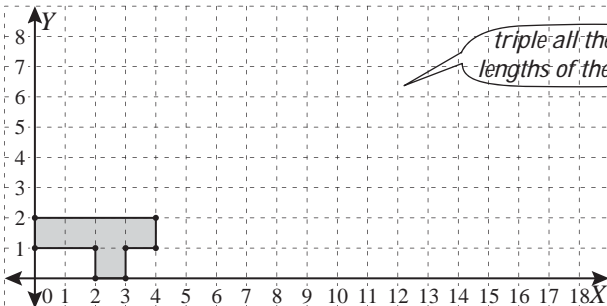
a) Find the scale factor of reduction for parallelogram ABCD.



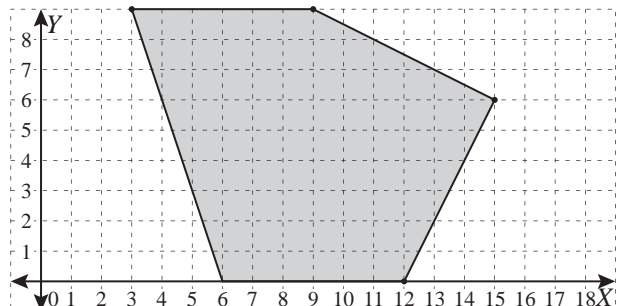
b) Find the scale factor of reduction for triangle DEF.



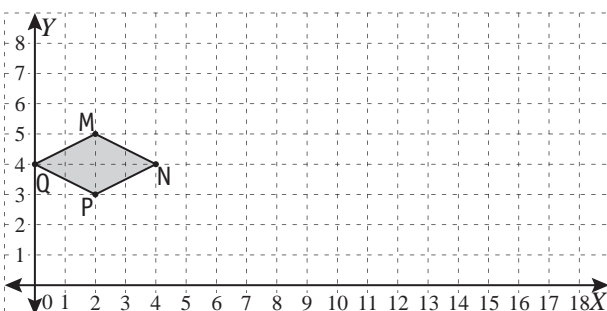
c) Redraw the shape enlarged by a scale factor of 3 about the origin of the axes.



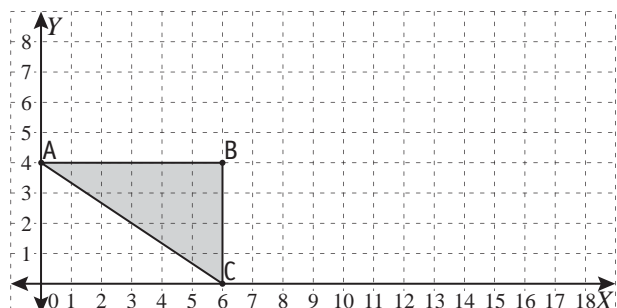
d) Redraw the shape reduced by a scale factor of 3 about the point of coordinates (6,0).



e) Redraw the rhombus MNPQ enlarged by a scale factor of 4 about point Q.

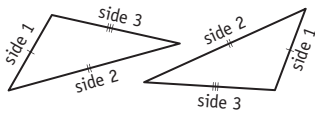


f) Redraw the triangle ABC enlarged by a scale factor of 2 about the origin of the axes. Label the enlargement A'B'C'. Are triangles ABC and A'B'C' congruent or similar?

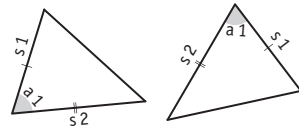


- Use the **tests for congruence** to check if two shapes are congruent (same size and shape).

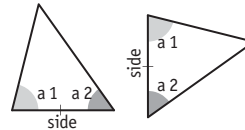
Side-side-side (**SSS**)



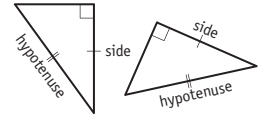
Side-angle-side (**SAS**)



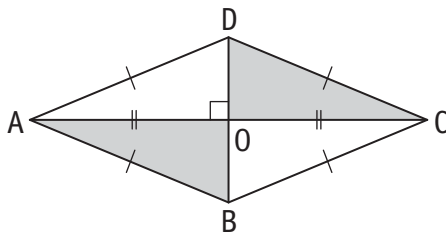
Angle-angle-side (**AAS**)



Right angle-hypotenuse-side (**RHS**)

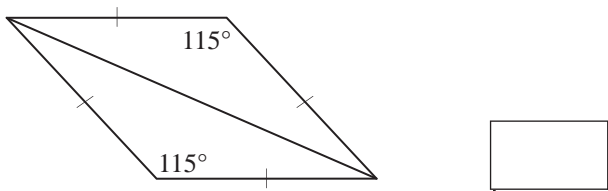


Q. Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that $\triangle AOB$ is congruent to $\triangle COD$?

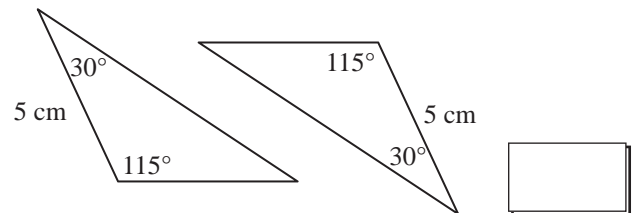


A. *Triangles AOB and COD are right-angled.*
 $AB = CD$ (hypotenuse)
 $AO = OC$ (side)
 $\Rightarrow \triangle AOB \cong \triangle COD$
 based on the congruence test
RHS

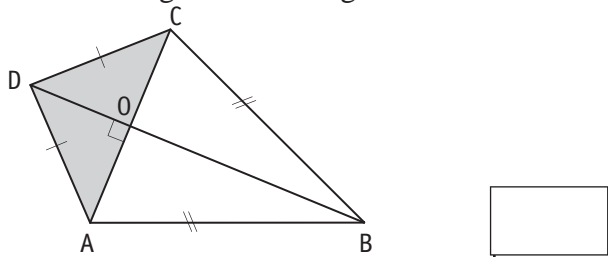
a) Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that these triangles are congruent?



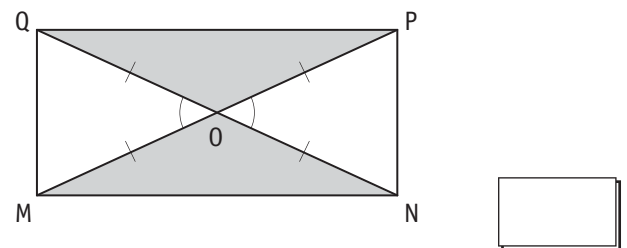
b) Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that these triangles are congruent?



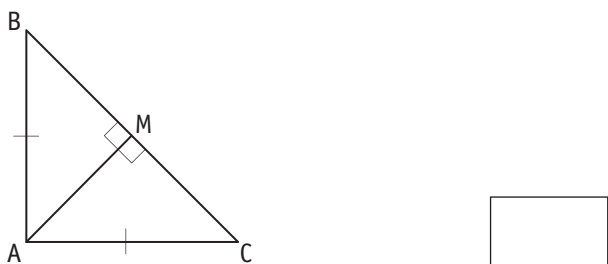
c) Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that triangle AOD is congruent to triangle COD?



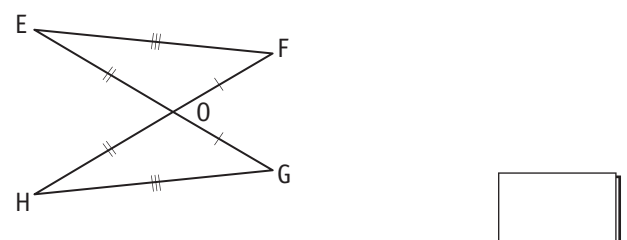
d) Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that triangle MON is congruent to triangle POQ?



e) Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that $\triangle AMB$ is congruent to $\triangle AMC$?



f) Which congruence test (SSS, SAS, AAS, RHS) can be applied to show that $\triangle EOF$ is congruent to $\triangle GOH$?



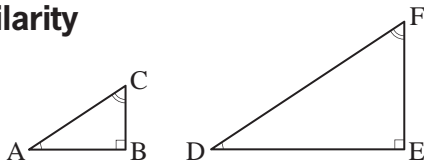
Skill 28.11 Recognising similarity of 2-dimensional shapes.

To find the value of an unknown side:

- Locate two pairs of corresponding sides; one pair should involve the unknown side.
- Set up the proportion (two equal ratios written as an equation).
- Solve the equation to find the value of the unknown.



similarity



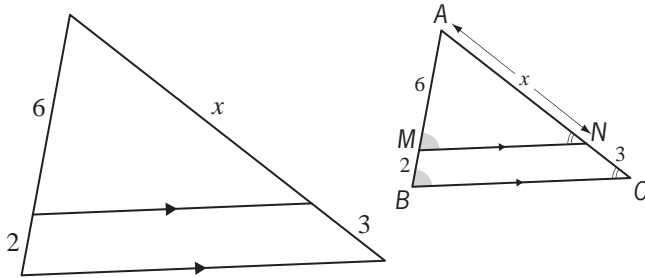
Corresponding angles are equal:

$$\angle A = \angle D \text{ and } \angle B = \angle E \text{ and } \angle C = \angle F$$

Corresponding sides are proportional:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Q. Find the value of x . [All measurements are in cm.]



A. Triangles AMN and ABC are similar because:

$\angle MAN = \angle BAC$ (the same angle)

$\angle AMN = \angle ABC$ (corresponding angles)

$\angle ANM = \angle ACB$ (corresponding angles)

The sides are proportional:

$$\frac{AM}{AB} = \frac{AN}{AC} \Rightarrow \frac{6}{8} = \frac{x}{x+3}$$

Cross multiply

$$6x + 18 = 8x$$

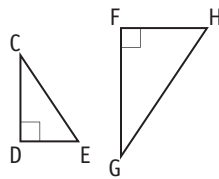
$$6x + 18 - 6x = 8x - 6x$$

Inverse of + 6x is - 6x

$$2x = 18$$

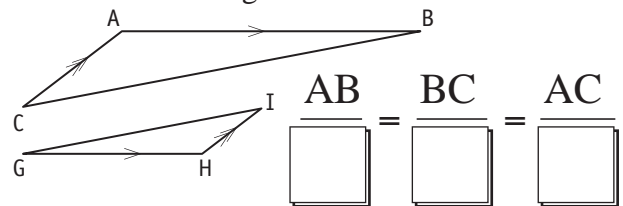
$$x = 9$$

a) Complete the pairs of equal angles for these similar triangles.

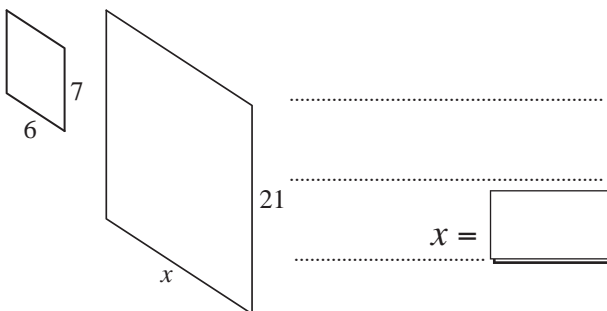


$$\angle C = \quad , \angle D = \quad , \angle E = \quad$$

b) Complete the ratios of corresponding sides for these similar triangles.

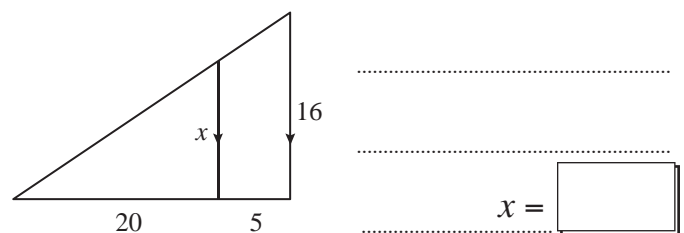


c) If these two parallelograms are similar, what is the value of x ? [All measurements are in cm.]



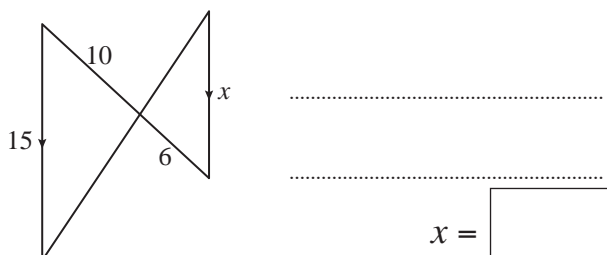
$$x = \boxed{\quad}$$

d) Find the value of x . [All measurements are in cm.]



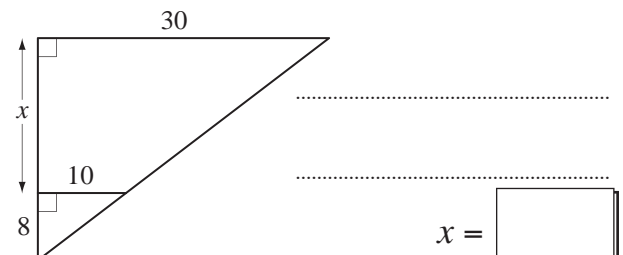
$$x = \boxed{\quad}$$

e) Find the value of x . [All measurements are in cm.]



$$x = \boxed{\quad}$$

f) Find the value of x . [All measurements are in cm.]



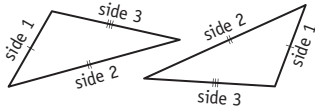
$$x = \boxed{\quad}$$

Skill 28.12 Identifying equal sides and angles to prove that two triangles are congruent.

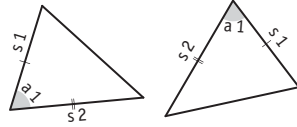
MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 3 4 4

- Find the equal sides or angles in the pair of congruent triangles (sides or angles which are in the same position in their respective triangles and are marked the same way).
- Use the **tests for congruence** to check if two shapes are congruent (same size and shape).

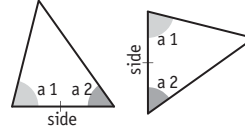
Side-side-side (SSS)



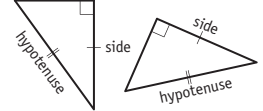
Side-angle-side (SAS)



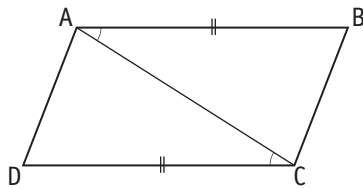
Angle-angle-side (AAS)



Right angle-hypotenuse-side (RHS)



Q. Write the pairs of equal sides and angles to prove that $\triangle ABC$ and $\triangle ACD$ are congruent. Which congruence test did you use?



A. The equal sides (in a matching position in the 2 triangles) are:

$$AB = CD$$

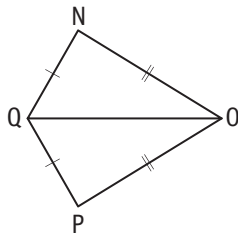
$$AC = AC \text{ (a common side)}$$

\Rightarrow The equal angles (in a matching position in the 2 triangles) are:

$$\angle BAC = \angle ACD$$

$\triangle ABC \cong \triangle ACD$, based on the congruence test **SAS**

a) Write the pairs of equal sides and angles to prove that $\triangle NOQ$ and $\triangle POQ$ are congruent. Which congruence test did you use?



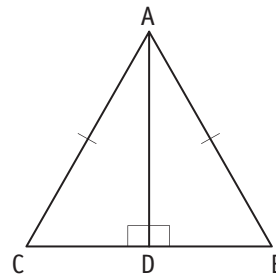
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congruence test:

b) Write the pairs of equal sides and angles to prove that $\triangle ACD$ and $\triangle ABD$ are congruent. Which congruence test did you use?



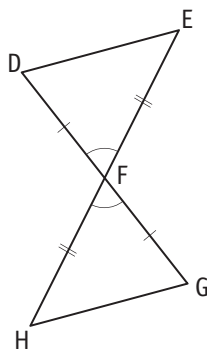
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congruence test:

c) Write the pairs of equal sides and angles to prove that $\triangle DEF$ and $\triangle FGH$ are congruent. Which congruence test did you use?



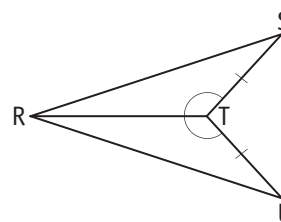
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congruence test:

d) Write the pairs of equal sides and angles to prove that $\triangle RST$ and $\triangle RTU$ are congruent. Which congruence test did you use?



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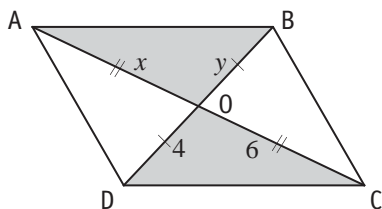
congruence test:

Skill 28.13 Using congruent triangles to find unknown sides and angles.

MM5.2 1 1 2 2 3 3 4 4
MM6.1 1 1 2 2 3 4 4

- Find the sides marked the same. They are equal in length.
- Find the angles marked the same. They are equal in size.
- Use the given values of sides and angles to find the unknown sides and angles.

Q. Find the value of x and y in this pair of congruent triangles. [All measurements are in cm.]

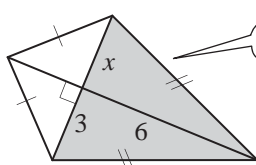


A. Triangles AOB and COD are congruent.

$$AO = OC = 6 \Rightarrow x = 6$$

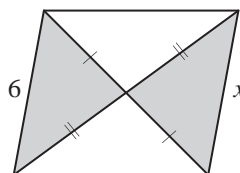
$$BO = OD = 4 \Rightarrow y = 4$$

a) Find the value of x given the pair of shaded triangles are congruent. [All measurements are in cm.]

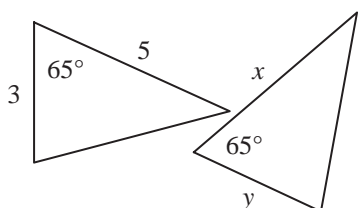


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b) Find the value of x given the pair of shaded triangles are congruent. [All measurements are in cm.]

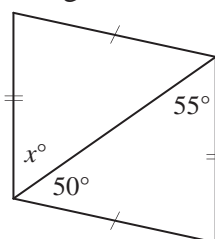


c) Find the value of x and y in this pair of congruent triangles.

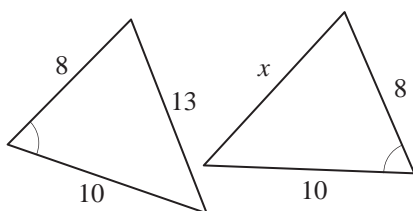


$$x = \quad y =$$

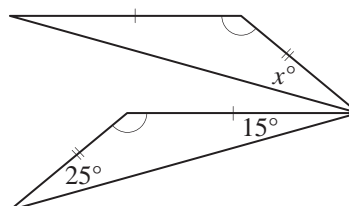
d) Find the value of x° in this pair of congruent triangles.



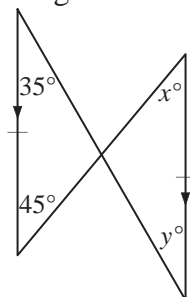
e) Find the value of x in this pair of congruent triangles. [All measurements are in cm.]



f) Find the value of x° in this pair of congruent triangles.

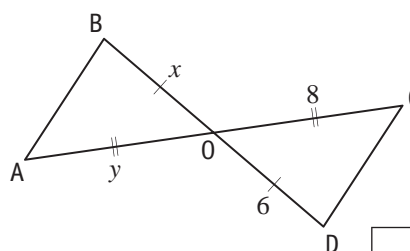


g) Find the value of x° and y° in this pair of congruent triangles.



$$x^\circ = \quad y^\circ =$$

h) Find the value of x and y in this pair of congruent triangles. [All measurements are in cm.]



$$x = \quad y =$$

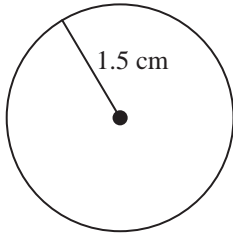
To redraw a shape to scale:

- Use a ruler and a set square to make the exact dimensions and proportions.
- Estimate the required side length, by comparing it to the other dimensions of the shape.

To find the scale ratio of a model:

- Convert the real-life dimension to the same unit of measurement of the model dimension.
- Divide the real-life dimension by the model dimension, to find the scale factor.
- Write the scale ratio of the model in the form of **1 : scale factor**.

Q. Determine the scale factor used when this circle represents the plan of a lake of diameter 600 m.



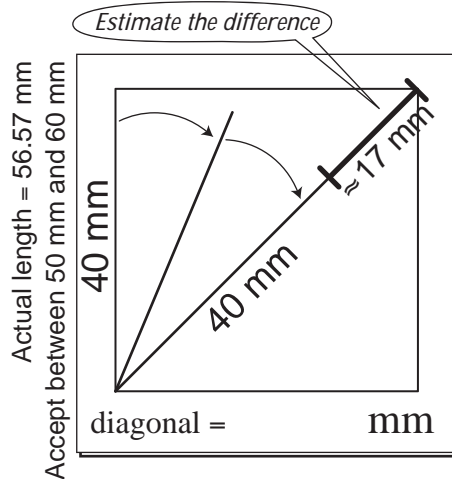
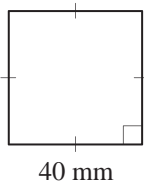
A. $600\text{ m} = 600 \times 100\text{ cm}$ 1 m = 100 cm
 $= 60\,000\text{ cm}$

$\text{scale factor} = 60\,000\text{ cm} \div 3\text{ cm}$
 $= 20\,000$

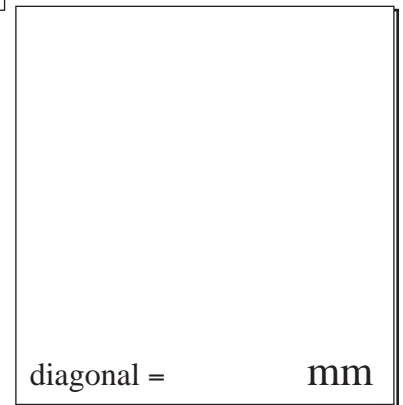
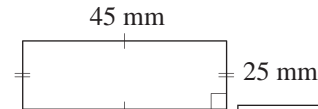
$\text{diameter} = 2 \times \text{radius}$
 $= 2 \times 1.5$
 $= 3$

$\text{scale ratio} = 1 : 20\,000$

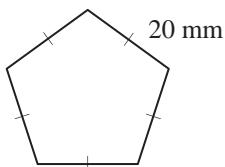
a) Redraw the square to scale and estimate the length of its diagonal.



b) Redraw the rectangle to scale and estimate the length of its diagonal.



c) Determine the scale factor used when this regular pentagon represents the plan of a bike track of perimeter 6 km.



$6\text{ km} = 6 \times 1\,000\,000\text{ mm}$

$= 6\,000\,000\text{ mm}$

$\text{scale factor} = 6\,000\,000\text{ mm} \div 100\text{ mm}$

= 1 :

d) Determine the scale factor used when this rectangle represents the plan of a football stadium of width 60 m.



$\text{scale factor} =$

= 1 :

